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# Deducing the general circulation from basic concepts and a few empirical facts

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## Abstract

The general circulation of the Earth's atmosphere is complex, linking a wide range of scales and physical processes. This special issue samples current research on a wide range of properties of the general circulation. This introductory article provides a context in which to place these articles by listing some principal balances such as: conservation of mass and momentum. Some implications and constraints are briefly described. The general circulation may be broadly deduced from those primary physical balances, dynamical constraints, and a few empirical facts (primarily the distribution of precipitation).

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## 1. Prologue

What is meant by the “general circulation”? For this special issue of Dynamics of Atmospheres and Oceans, we define the general circulation to be: large scales (everything >1000 km), persisting a season or longer, *and*, all the processes necessary to sufficiently explain (or directly maintain) these large scale, persistent circulations. What follows the ‘*and*’ in that definition greatly widens the definition to include other variables than just wind. Solely using zonal average fields would not work—one needs zonally varying fields to maintain (and thus explain) the zonal mean, as has been demonstrated numerous times.

The question of *why* there should be a general circulation is relatively easy. The atmosphere develops a circulation in an attempt to reach a thermal equilibrium between *net* sources and sinks

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29 of radiative heating and cooling. Consequently, the solar radiation absorbed has greater latitudinal  
30 variation than does the terrestrial radiation back to space (see Fig. 1a).

31 Of course neither radiative pattern is completely symmetric with respect to longitude or latitude.  
32 (See Fig. 1b and c. While these data have biases, they are easily sufficient to illustrate the qualitative  
33 discussion here.) The general pattern is partly explained by surface properties and the geometry of  
34 a rotating Earth, tilted with respect to the plane of the ecliptic. As much, if not more of the pattern  
35 seems driven by atmospheric circulations that transport heat, cause variable radiative properties  
36 (clouds, snow cover, etc.), and absorb and emit radiation. Oceanic circulations and topography  
37 further complicate the atmospheric circulation.

38 Despite the great complexity of the atmospheric and oceanic processes, one can deduce the  
39 general properties of the large scale atmospheric circulation by applying some balances augmented  
40 by a few empirical facts.

## 41 2. Deducing the general circulation

42 By exploiting the concept of balances, we can deduce much about the general circulation.  
43 Certain balances are believed to hold and these give rise to formulas and understanding as to why  
44 the observed circulation looks like it does.

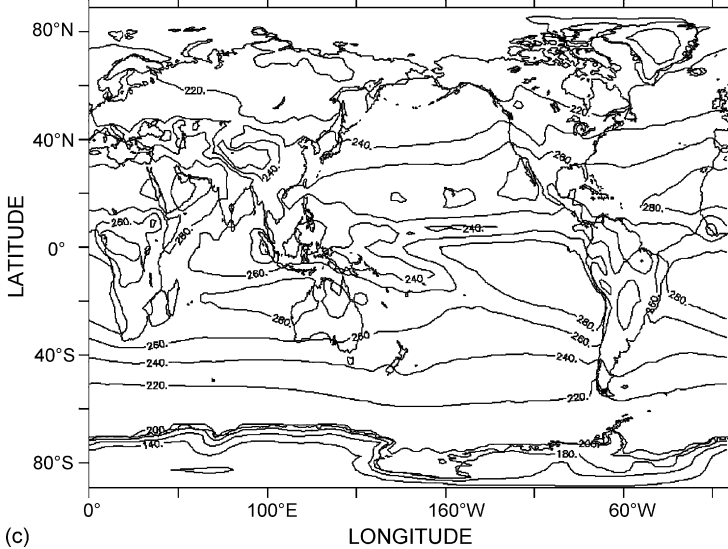
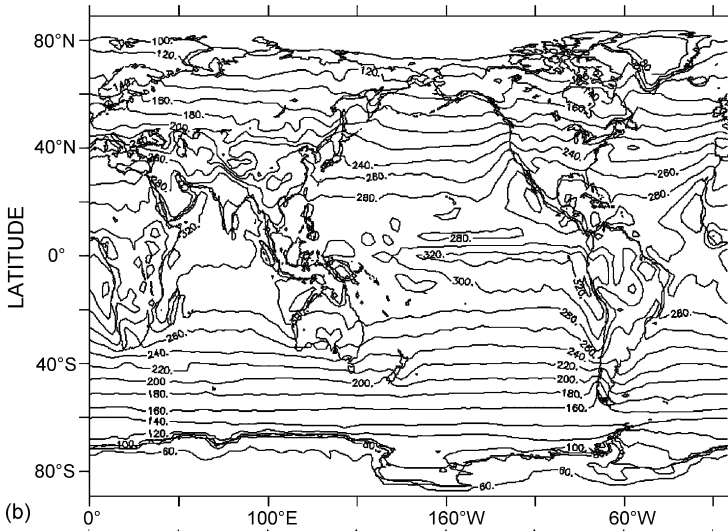
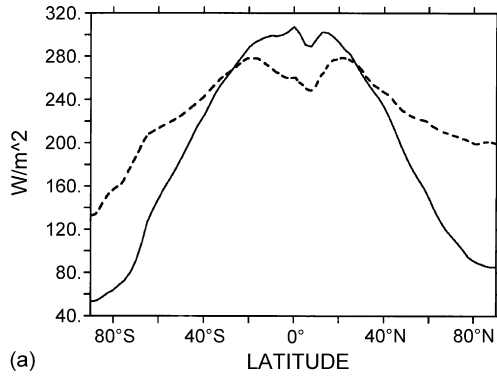
45 One can invoke physical concepts; two examples being: radiative and angular momentum  
46 balance. Radiative balance states that when averaged over the surface of the globe, there will be  
47 no net heating up (or cooling down) of the Earth if the same amount of energy is emitted as is  
48 absorbed. While there appears to be evidence for global warming, that rate of temperature increase  
49 is still far less than the seasonal cycle at most places on Earth. Angular momentum balance has  
50 several implications for the general circulation. One implication is surface winds need a mixture  
51 of easterlies and westerlies so as not to apply a net torque upon the solid Earth. As a trivial  
52 example, if the surface winds were westerly everywhere, then the atmosphere would be applying  
53 a net torque upon the oceans and solid Earth, as one consequence the rotation of the Earth would  
54 speed up.

55 One can use balances, physical reasoning, and the observed precipitation to deduce some basic  
56 properties. Various laws, such as ideal gas and conservation of potential temperature for adia-  
57 batic motions may be applied. Mathematical simplifications, such as hydrostatic and geostrophic  
58 balances are useful. An overarching assumption shall be that the atmosphere can be considered  
59 annually stationary: any trends that remain when fields are averaged over a year are small and can  
60 be ignored for this discussion.

### 61 2.1. Radiation and temperature

62 Excluding global warming, the Earth does not heat up in the net over a year. So, one may  
63 expect a balance between absorption and emission of radiant energy. Absorption is complicated  
64 by variations in the reflectance, absorbance, and transmittance of various terrestrial constituents.  
65 Despite these complications, the orbital geometry is a primary cause for more absorption averaged

Fig. 1. Global radiative balance for the Earth. (a) Zonal and time average radiation at the top of the atmosphere. Solid line: incoming, solar, absorbed radiation; dashed line: upwelling, longwave radiation from the Earth. (b) Time average incoming, solar, absorbed radiation at the top of the atmosphere. (c) Time average upwelling, longwave radiation from the Earth. ECMWF Reanalysis 1 (ERA-15) monthly mean data from January 1979 through December 1993. Contour interval 20 W/m<sup>2</sup> in b and c.



66 over the tropics than averaged over polar regions. This fact creates a meridional temperature  
67 gradient. The loss of radiation back to space is complicated by several factors; for example,  
68 the infrared emission from the Earth is not from a single surface, but includes all levels in the  
69 atmosphere as well as the ocean and Earth's surface.

70 If the rate at which the atmosphere cools down by infrared emission were large compared to the  
71 temperature of the Earth divided by the length of a day, then the primary temperature gradient of the  
72 atmosphere would be from the subsolar point to the shadow side of the planet. Radiative cooling  
73 rates in the troposphere are around  $2^\circ$  per day, which is much less than the roughly 210–310 K  
74 temperatures of the troposphere. (This thermal inertia is largely maintained by radiative emission  
75 from the Earth's surface.) As a consequence, the diurnal cycle of temperature is small compared  
76 to the daily mean temperature. Also, the terrestrial longwave emission is similar to the long term  
77 average (Fig. 1c). Similarly, the 20–70 °C range in temperature observed from equator to pole  
78 in the troposphere exceeds the temperature change from midday to midnight. (The equator to  
79 pole temperature range varies with elevation in the atmosphere.) So, the temperature gradient  
80 is primarily meridional rather than zonal. And, longer term (seasonal) average differences in  
81 radiation absorbed and emitted therefore dominate the temperature distribution.

82 Defining net radiation as the local difference of absorption minus emission, net radiation  
83 is positive over much of the tropics, and negative in the polar regions (averaged over a year).  
84 This pattern of net radiation implies a meridional transport of heat, perhaps by some meridional  
85 motions. Such motions could conceivably remove the implied meridional temperature gradient,  
86 but if they did, the motions would also remove their driving mechanism.

87 In the stratosphere the temperature pattern is a bit different. Stratospheric temperatures are  
88 dominated by radiative and transport processes, whereas tropospheric temperatures are also influ-  
89 enced by latent heat release, and exchanges with the Earth's surface. The orbital tilt and spherical  
90 geometry of the Earth combine to give a rather flat distribution of incoming solar radiation in the  
91 high summer latitudes. Hence, temperatures have little meridional gradient in much of the upper  
92 stratosphere in mid-summer. At upper stratospheric levels, temperatures tend to be coldest near  
93 the winter pole. In the lower stratosphere, the coldest temperatures are technically in the tropics,  
94 owing to the much higher tropopause there than in polar regions. However, in mid and upper  
95 stratosphere elevations, temperatures tend to decrease from the summer polar region to the winter  
96 polar region during those seasons. In other seasons, the mid and upper level temperatures tend to  
97 decrease from the tropics towards both poles.

## 98 2.2. Pressure and geopotential

99 Both water vapor content as well as temperature in the troposphere decrease from tropics to the  
100 polar regions. Hence, the virtual temperature decreases in similar fashion. From the hypsometric  
101 equation, the meridional virtual temperature gradient thereby creates a meridional geopotential  
102 gradient. The meridional temperature gradient applies for each layer (above the boundary layer,  
103 say) in the troposphere so the thickness variation in each layer is additive. Thus, the meridional  
104 gradient of geopotential increases with elevation. Near the ground, the pressure pattern has com-  
105 parable longitudinal and latitudinal variations. However, the pressure at elevations above roughly  
106 2 km (and geopotential height of pressure surfaces for  $P < 850$  hPa) also decrease from tropics to  
107 polar regions.

108 By similar reasoning, in the stratosphere, the meridional gradient of geopotential decreases  
109 with increasing elevation in the summer hemisphere and tends to increase with elevation in the  
110 winter hemisphere.

111 Where there is a horizontal gradient of pressure there will be a pressure gradient force (PGF).  
112 In the troposphere the PGF is directed towards the poles and provides the impetus to initiate the  
113 circulation.

### 114 2.3. Winds

115 The Earth rotates such that observed velocities outside the tropics are subject to a Coriolis force  
116 comparable to the poleward PGF. Hence, the meridional gradient of pressure (and geopotential)  
117 implies westerly winds from geostrophic balance. Since the gradient increases with height then so  
118 must the westerly winds in the troposphere. It was established above that heat must be transported  
119 poleward to explain the latitudinal distribution of net radiation. If the heat is transferred in a way  
120 that employs meridional motion, then zonal winds are created to conserve angular momentum  
121 during such meridional motions. Are the zonal winds implied by the virtual temperature distribu-  
122 tion consistent with zonal winds created by large scale angular momentum conserving meridional  
123 motions? Presumably a balance can be struck whereby stronger meridional motions more effec-  
124 tively remove the equator to pole temperature gradient thereby lessening the zonal thermal winds,  
125 and vice versa.

126 Conservation of angular momentum by meridional motions can create very large zonal winds.  
127 For example, if an air parcel having zero zonal motion at the equator were brought to 30N  
128 latitude, an angular momentum conserving zonal wind,  $U_{amc}$  would equal 134 m/s. This is twice  
129 the time mean speed of the strongest portion of the subtropical jet stream found at a similar latitude.  
130 Obviously, one can reduce this acceleration by internal viscosity and vertical mixing of momentum  
131 by turbulence or convection. The greater the length of time over which friction and mixing can  
132 act, the greater the reduction possible. For realistic estimates of internal friction, *meridional*  
133 wind speeds of a few cm/s keep *zonal* winds within observed bounds for motions spanning the  
134 tropics and subtropics (Grotjahn, 1993; Section 6.2.3.2) In the tropical upper troposphere observed  
135 meridional velocities average 5–10 times larger, suggesting that either the friction is too weak  
136 and/or other mechanisms are important. Another mechanism might be convection that mixes  
137 momentum vertically. This may be happening within ‘tropical plumes’ created in advance of  
138 upper level troughs that penetrate deeply into the tropics; air is accelerated poleward, but actively  
139 growing clouds moderate the angular momentum by vertical mixing.

### 140 2.4. Heat transport and mass balance

141 Surface pressure values in polar regions do not have a significant trend. So, a meridional  
142 circulation that transports heat poleward would be expected to have compensating equatorward  
143 motion, otherwise mass would accumulate in the polar region.

144 The previous section suggests that meridional velocities are small based on observations and  
145 angular momentum considerations. Can such small meridional motion still transport sufficient  
146 heat? This question introduces mass and/or temperature field considerations. If the air is moving  
147 both poleward and equatorward, then net sensible heat transport depends on the difference in dry  
148 static energy (DSE) between the north and southward moving air. (Latent heat processes from  
149 water evaporation, transport, and condensation are being ignored for the moment.) Consequently,  
150 to have the same net heat flux, a smaller meridional velocity must be balanced by a larger difference  
151 in DSE between the north and southward moving air. If the larger DSE difference were due to  
152 temperature changes, then that means larger static stability. Alternatively, if the vertical separation  
153 were made larger between opposing meridional motions then the DSE difference would be larger

154 and the meridional motions need not be as strong. So in theory such small meridional velocities  
155 can transport heat if the DSE change is large enough. Clearly the DSE variation depends on the  
156 temperature variation, both horizontally and vertically.

157 The DSE will increase with height for a statically stable atmosphere. If zonally and vertically  
158 integrated mass transport in the meridional is to be zero, then a poleward DSE heat flux occurs  
159 when the poleward moving air has higher DSE than the equatorward moving air. Hence, if the upper  
160 level flow has larger DSE than the low level flow, the upper level flow would need to be poleward.

161 Water mass conservation is a different issue because water exists in three different states on  
162 Earth. Water could transport heat poleward if it were evaporated in lower latitudes (removing  
163 heat there), transported poleward by the winds, and condensed at higher latitudes (releasing the  
164 heat there). The mass of atmospheric water in vapor form would not be conserved. Water could  
165 conceivably be condensed, fall as precipitation, and run off, returning to the lower latitude in  
166 liquid form. While this may seem like a reasonable way to accomplish the heat transport, a major  
167 drawback is that this mechanism works against the dry heat transport by a zonally averaged  
168 circulation. To see this, one defines a moist static energy (MSE) by adding latent heat content to  
169 the DSE. The scale height of water vapor ( $\sim 2$  km) is much smaller than that of dry air ( $\sim 8.5$  km).  
170 So for high enough moisture content, the moist static energy could conceivably decrease with  
171 height and if so, poleward heat transport would be accomplished if the poleward moving air were  
172 at *low* levels—opposite to the pattern deduced for DSE. In much of the tropics, MSE *is* observed  
173 (e.g. Gustafson and Weare, 2004) to decrease with height in the lower troposphere ( $P > 700$ ). It  
174 is conceivable that a circulation with surface poleward motion and mid to lower tropospheric  
175 equatorward return flow might transport the necessary heat. Such a circulation might also have a  
176 lot of boundary layer friction to control the development of excessive zonal winds from angular  
177 momentum conservation. There are potential problems with this idea. First, a mechanism (vertical  
178 mixing or solar absorption perhaps) may be needed to overcome a meridional gradient of MSE that  
179 may oppose the motion. Second, it may conflict with the observed tropical precipitation pattern.

180 The observed distribution of precipitation is crucial to deducing the Earth's general circulation.  
181 The primary maximum in precipitation is near the equator and relative minima occur in the  
182 subtropics; this pattern eliminates the zonal mean meridional 'boundary layer' cell postulated in  
183 the previous paragraph. Furthermore, secondary maxima of precipitation in northern and southern  
184 middle latitudes must be explained and that selects for a different midlatitude circulation type  
185 than in the tropics.

## 186 2.5. *Surface torques*

187 When wind blows across the surface of the Earth friction slows down the wind while imparting  
188 a torque upon the Earth. If the surface winds were to blow the same direction as the rotation  
189 everywhere on a planet with no topographic features, then the winds would be accelerating  
190 the planetary rotation. Such reasoning suggests that surface winds on the Earth are unlikely to  
191 be westerly (or easterly) everywhere. The simplest conclusion might be that there is a balance  
192 between the torque applied in regions of surface easterlies versus the torque applied by regions  
193 of surface westerlies.

194 Surface torques are applied not only by winds blowing across the surface of the Earth but  
195 also by surface pressure differences across mountain ranges. Restricting our attention to zonal  
196 flows, then westerly winds impart an eastward torque upon the Earth. A surface pressure pattern  
197 of low on the windward side and high on the lee side of a topographic feature imparts a "mountain  
198 torque" that opposes the torque by the winds.

199 How much of a surface pressure difference across the mountain is needed to balance a given  
200 zonal wind? Is such a pressure difference “realistic”? The Earth has several major mountain ranges  
201 that may provide such a balance, of particular interest for westerly winds are the north–south  
202 oriented ranges such as the Andes and Rockies. To gain a sense of the magnitudes involved, it  
203 is instructive to make a crude calculation that balances surface stress  $\tau$  from the winds against  
204 mountain torque. Specifically, for the Andes, the balance (see Peixoto and Oort, 1992; Section  
205 11.1.4) is roughly:

$$206 \int_0^{2\pi} \int_{-\pi/3}^0 \tau a^3 \cos^2(\varphi) d\varphi d\lambda \int_0^{z_p} \int_{-\pi/3}^0 \{P_E - P_W\} a^2 \cos(\varphi) d\varphi dz \quad (1)$$

207 where  $a$  is the Earth’s radius ( $\sim 6370$  km),  $\varphi$  the latitude,  $\lambda$  the longitude,  $z_{\text{top}}$  an elevation above  
208 the highest point of the Andes,  $P_E$  the surface pressure on the east side of the Andes, while  $P_W$   
209 the corresponding pressure on the west side. (Where  $z$  exceeds the local height of the Andes, the  
210 pressure difference is zero.) Other Southern Hemisphere topography is ignored. Imagine surface  
211 winds of the Southern Hemisphere that are a uniform 10 m/s over all longitudes and over latitudes  
212 from equator to 60S. Further assume the pressure difference is uniform with elevation and latitude  
213 and important for the lowest 3 km. Approximate the surface stress using a drag coefficient of  $10^{-3}$   
214 and average surface density of  $1.25 \text{ kg m}^{-3}$  such that  $\tau \sim 1.25 \times 10^{-1} \text{ kg m}^{-1} \text{ s}^{-2}$ . The resultant  
215 pressure difference  $dP (= P_E - P_W)$  is  $\sim 24$  hPa. Such a pressure change is within the range of  
216 sea level pressure observed and therefore is not unrealistically large. Unfortunately, the observed  
217 sea level pressure change across most of the Andes has the opposite sign, implying that surface  
218 pressure at other Andean elevations might not be able to balance surface westerlies. (Note: such  
219 westerlies would exert stress upon the ocean, causing a zonal change in the height of sea level,  
220 but the change would largely pass the wind torque along to the solid Earth.) Another problem is  
221 westerlies over the Andes would be up, not down the sea level pressure gradient.

222 To summarize, the discussion of balances and key empirical facts implies the following six  
223 properties:

- 224 (i) Incoming radiation makes the tropics warmer than the poles, which in turn creates an equa-  
225 torward pressure gradient (and hence poleward pressure force) above some level in the  
226 atmosphere.
- 227 (ii) The pressure gradient and Earth’s rotation lead the  $T$  gradient to have associated westerly  
228 winds that increase with elevation.
- 229 (iii) Net radiation implies that there must be poleward transport of heat.
- 230 (iv) Mass balance requires the poleward heat transport to have “equal” mass components north  
231 and south.
- 232 (v) Angular momentum balance anticipates “roughly equal” areas of easterly and westerly sur-  
233 face winds; and/or high pressure on the lee side and low on the windward side of major  
234 north–south oriented mountain ranges.
- 235 (vi) Water mass balance and transport are needed to explain precipitation maxima in the midlat-  
236 itudes and tropics.

### 237 3. Thought experiments

238 To help understand the context of what follows in this special issue, it is useful to have an  
239 idea of what circulations one might expect. One can make quite a bit of progress by performing

240 a series of “thought experiments”. The discussion that follows is qualitative, but it expands and  
241 makes visible the quantitative principles listed above. Some candidate, “imaginary” circulations  
242 are proposed until one qualitatively satisfies these six properties discussed above.

243 3.1. Idea #1: pure northward motion

244 While this motion transports sensible heat poleward, by bringing warm air from the tropics  
245 towards the poles, it has already been rejected above due to a lack of mass conservation. Fur-  
246 thermore, angular momentum is not balanced, unless the motion were heavily slowed by friction.  
247 There is no mechanism to create the observed precipitation pattern.

248 3.2. Idea #2: alternating meridional flows

249 In an effort to solve the mass balance problem of idea #1, one might propose meridional  
250 circulations that alternate between northward and southward motion (Fig. 2a) through the depth of  
251 the troposphere (or perhaps even more of the atmosphere). The arrows shown in Fig. 2a are trajec-  
252 tories. Heat is now mixed both directions, cooling tropical regions directly while warming polar  
253 regions. Mass balance is satisfied by matching the mass of poleward and equatorward moving air.

254 Unfortunately, angular momentum balance might require slow meridional motion to occur  
255 mainly within a boundary layer. The motion above the boundary layer would be strongly ‘tilted’  
256 in the zonal direction as shown in Fig. 2a. In order to avoid a net torque upon the Earth, the strong  
257 westerlies (from angular momentum conservation) in high latitudes would be balanced by strong  
258 easterlies in low latitudes. If there is vertical mixing of momentum (by convection, if not a large

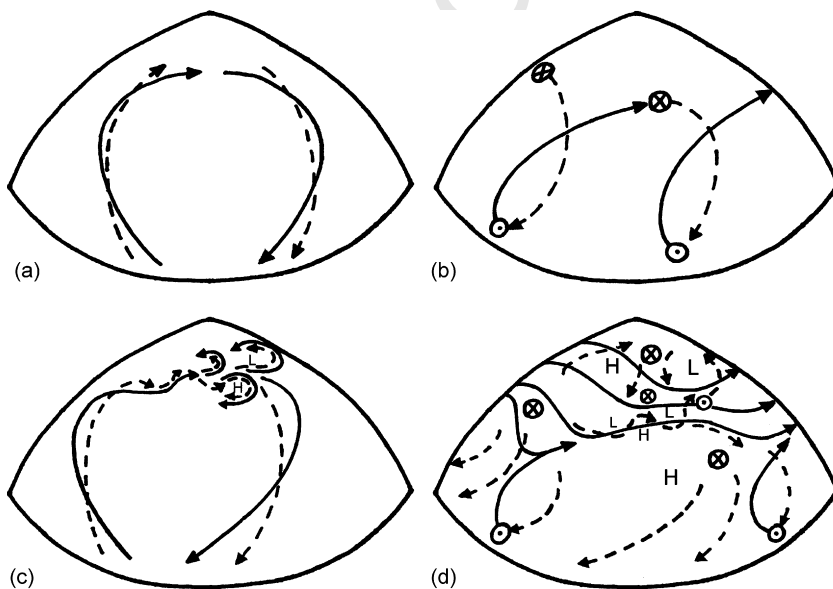


Fig. 2. Schematic diagrams of selected candidate circulations generally showing proposed air parcel trajectories. Solid lines show upper level flow, dashed lines show low level flow. Circles enclosing a dot suggest upward motion; circled  $\times$  for downward. (a) ‘Idea #2’ alternating flows; (b) ‘idea #3’, ‘one big Hadley cell’; (c) ‘idea #4’ barotropic eddies; (d) ‘idea #6 hybrid’ circulation that combines baroclinic eddies in high latitude with a ‘Hadley’ circulation in low latitudes.



259 scale overturning) then the winds are somewhat moderated. This candidate circulation would  
260 develop large upper level winds similar to the jet streams, however, it has no obvious mechanism  
261 by which to limit the jet to subtropical or midlatitude locations.

262 This candidate would have trouble explaining other important observed features. The midlat-  
263 itude precipitation maximum might be explainable as follows. The poleward moving air in the  
264 boundary layer might be quite moist, as that air migrates poleward it cools (by net radiation, and  
265 by surface exchange) to a point where drizzle might occur. Explaining the midlatitude precipita-  
266 tion maxima as drizzle is a bit of a stretch! This scheme also has no obvious explanation for the  
267 tropical precipitation maximum. Finally, the predicted strong upper level equatorial easterlies are  
268 really not seen; the closest phenomenon being the Indian Ocean monsoon jet.

269 While this circulation is rejected, it does suggest trying alternating motions in the vertical and  
270 in the horizontal.

### 271 3.3. Idea #3: “one big Hadley cell”

272 A circulation having alternating motion in the vertical is the “Hadley cell”. The Hadley cell  
273 circulation could be similar at all longitudes and form a loop: with rising in the tropics, poleward  
274 motion in the upper troposphere, sinking at higher latitudes, and equatorward motion in the  
275 lower troposphere. This Hadley cell is depicted in Fig. 2b. The schematic trajectories depict how  
276 poleward motion has westerly wind acceleration while equatorward motion in a boundary layer  
277 (subject to greater frictional dissipation) has less easterly acceleration. If mountain torques are  
278 absent, the wind might have no net torque upon the Earth’s rotation by a careful adjustment of  
279 where the location lies of zero surface zonal wind. Mass is conserved by matching northward and  
280 southward mass fluxes. Heat transport is accomplished if the poleward flow has higher MSE than  
281 the equatorward flow. This is possible if the poleward flow is at tropospheric levels high enough  
282 so that the gravitational potential in MSE overcomes any high moisture content at low levels.  
283 Another reason for having a high troposphere poleward flow is to develop strong westerlies at  
284 jet stream level. The rising motion in the equatorial region provides an obvious explanation for  
285 the equatorial precipitation maximum (though details are missing like overcoming the vertical  
286 gradient of MSE within thunderstorm ‘hot towers’).

287 Some observed features are problematic. If this ‘big Hadley cell’ extends to polar regions, how  
288 would the jet streams occur where they do and not much further poleward? The zonal wind speeds  
289 reached would be unrealistically large in mid- and higher latitudes. Finally, there is no obvious  
290 explanation for the observed midlatitude precipitation maximum.


291 So while this circulation has promise in the tropics, it cannot explain all the general circulation.

### 292 3.4. Idea #4: barotropic eddies

293 The alternating meridional motions of Fig. 2a lead to very large horizontal wind shears. At some  
294 point, large shears may become hydrodynamically unstable. A simple approximate determinate  
295 for (barotropic) instability is that the meridional gradient of absolute vorticity  $Q$ , change sign in  
296 the domain. In spherical geometry (Baines, 1976)  $Q$  becomes:

$$297 \quad Q = \beta - \frac{1}{a} \frac{\partial}{\partial \varphi} \left\{ \frac{1}{a \cos(\varphi)} \frac{\partial}{\partial \varphi} (U \cos(\varphi)) \right\} \quad (2)$$

298 where  $a$  is the Earth’s radius,  $\varphi$  the latitude,  $\beta$  the meridional gradient of the Coriolis parameter. ( $Q$   
299 is the absolute vorticity gradient for a purely zonal flow, the contribution by meridional motion to  $Q$

300 has been ignored.)  $\beta$  decreases while the horizontal shear of  $U$  increases for increasing magnitude  
301 of latitude. However, since shear increases with latitude, the relative vorticity of such a zonal flow  
302 is negative. For the Earth, a zonal flow constructed from  lar momentum conservation would  
303 have  $Q=0$  at all latitudes! This result is a consequence that flow also conserves absolute vorticity.  
304 If some internal viscosity is added, then  $Q<0$  at all latitudes and the flow is stable to barotropic  
305 eddies according to classical reasoning, even if large horizontal shear is still present. But these  
306 assumptions are too simplistic.

307 If topography is added, then waves of a scale similar to the topography may be forced and  
308 these would interact and grow, releasing some horizontal shear instability. Also, viscosity may  
309 lead to form drag instability (e.g. Frederiksen and Frederiksen, 1990). If moisture is included,  
310 perhaps through moist convective heating, then the condition that  $Q$  change sign may be “neither  
311 a necessary nor a crucial condition for instability for a barotropic flow” (Mak, 1983; p. 2355). In  
312 short, it may be that barotropic eddies could form, by several different mechanisms.

313 This idea (Fig. 2c) might appear to transport heat poleward by motion in both directions  
314 (warm air poleward, cold air equatorward). However, the colder air would have smaller thickness  
315 (between isobaric surfaces) which in turn creates zonal geopotential gradients which in turn  
316 creates geostrophic winds. The consequence of those winds is unclear, perhaps the whole pattern  
317 migrates westward, perhaps it breaks down into multiple eddies.

318 The scheme can satisfy mass balance. A mean horizontal zonal velocity can be added to the flow  
319 in such a way as to create compensating amounts of easterly and westerly surface torque. Friction  
320 in the boundary layer of barotropic lows implies low level convergence with consequent rising  
321 motion that may lead to precipitation, though it is unclear how this gets focused to midlatitudes  
322 except by restricting eddies to higher latitudes where some threshold of horizontal shear is crossed.

323 This idea shows some promise since it recognizes that hydrodynamic instability can play a  
324 role. However, there are some critical flaws. First, how the heat transport occurs is unclear. In of  
325 themselves, barotropic eddies do not transport heat. However, one might argue that radiative gain  
326 (loss) in the tropical (polar) regions leads to a net heat flux if the resident time in each region  
327 is long enough for sufficient radiative temperature changes. Second, the energetics of such lows  
328 would be a bit strange; while radiation-induced temperature changes drive meridional motion that  
329 creates zonal wind shear that is unstable to barotropic eddies, kinetic energy is lost by friction and  
330 conversion to eddy potential energy. Third, a mechanism for the tropical precipitation maximum  
331 is missing.

### 332 3.5. Idea #5: vertically tilted eddies

333 The barotropic eddies thought experiment invoked instability of horizontally sheared flow.  
334 The meridional temperature gradient requires vertical shear if thermal wind balance holds. As for  
335 horizontal shear, there is a classical necessary condition for instability of vertically sheared flows.  
336 Reasoning by analogy to Cartesian geometry, an appropriate governing equation for stability of a  
337 zonal mean flow may be deduced from a potential vorticity conserving governing equation (e.g.  
338 Grotjahn and Castello, 2002). Using a streamfunction ( $\Psi$ ) to represent the zonal mean flow, then

$$339 \frac{\partial}{\partial \mu} \nabla^2 \Psi + 4\mu^2 \frac{\partial}{\partial \mu} \left\{ \frac{\partial}{\partial p} \left( \frac{p}{\sigma} \frac{\partial \Psi}{\partial p} \right) \right\} + 2 + \text{Boundary Contributions} = Q_y \quad (3)$$

340 where  $\mu$  (= sine of latitude),  $p$  the vertical coordinate pressure and static stability, and  $\sigma$  can vary  
341 in the vertical. Boundary Contributions refers to contributions to  $Q_y$  from the bottom and top  
342 boundaries. In Cartesian geometry a change in sign of the mean flow potential vorticity gradient

343 is necessary to have unstable waves (Charney and Stern, 1962); one might reasonably extend  
344 this classical analysis to the corresponding  $Q_y$  in spherical geometry. Simplifying the schematic  
345 circulation to be described by flow in two tropospheric layers, Baines and Frederiksen (1978)  
346 extend the Phillips (1954, Cartesian geometry) condition for stability to spherical geometry as:

$$347 \quad U_U - U_L \geq \frac{622.5 \cos(\varphi)}{a\Omega \sin^2(\varphi)} (\Theta_U - \Theta_L) \quad (4)$$

348 where the subscripts refer to upper and lower layers.  $U$  is the zonal mean flow, while  $\Theta$  is the  
349 horizontal mean potential temperature. It is clear that this criterion decreases nonlinearly with  
350 latitude. Static stability helps control the latitude at which the instability criterion is exceeded in  
351 two ways. The greater the static stability then: (i) more vertical shear is needed for instability but  
352 (ii) the vertical difference of DSE is larger. The heat transport is more efficient (smaller meridional  
353 motion is needed to transport the same amount of heat). But smaller meridional motion reduces  
354 the strength of the  $U_{\text{amc}}$  (by more time for friction to act).

355 If zonal mean flow has vertical shear from meridional motion that conserves angular momentum  
356 (zonal wind equals  $U_{\text{amc}}$ ) at upper levels, but is strongly damped by friction at low levels, then  
357 one might approximate the shear as some fraction of  $U_{\text{amc}}$ . For illustrative purposes, if the shear  
358 is 70% of  $U_{\text{amc}}$  and the potential temperature difference from 750 to 250 hPa is 30 K (roughly the  
359 observed value in the subtropics) then the instability criterion is met for latitudes higher than  $32^\circ$ .

360 The amplifying waves feed upon the vertical shear and thereby provide a mechanism to control  
361 the unrealistically large zonal velocities created by a purely angular momentum conserving flow.  
362 The resultant waves will be tilted against the vertical shear. That can occur if the temperature and  
363 mass fields are offset, and if so, there will be a net heat flux across the mean wind, i.e. poleward.  
364 The needed poleward heat transport is accomplished by mixing cold air equatorward and warm  
365 air poleward. The motion around the eddies can satisfy mass balance and it is also possible to  
366 balance surface westerly and easterly torques. The poleward heat fluxes are accompanied by  
367 vertical motions across isentropes; such that warm air rises to a higher potential temperature  
368 surface. Since such warm air is expected to have moisture, the amplifying waves will also have  
369 areas where condensation and precipitation are likely to occur, thus providing a new mechanism  
370 for the midlatitude precipitation maximum. Surface friction further leads to low level convergence  
371 in the troughs of such waves, further amplifying the midlatitude precipitation maximum.

372 This idea seems to work well in middle and high latitudes. The crudely estimated stability  
373 criterion suggests that such baroclinic waves are not likely to be the main factor in the tropical  
374 latitudes. That result, coupled with the ability of the Hadley cell to explain tropical features  
375 suggests that the tilted eddy and Hadley cell ideas be applied in different latitude bands with some  
376 sort of matching in between.

### 377 3.6. Idea #6: hybrid circulation

378 The preceding discussion suggests combining ideas #3 and #5. The challenge is to sort out  
379 how the two quite different ideas can be matched. This idea is more difficult to draw than the  
380 other candidates since the middle and high latitude baroclinic eddies are traveling and they change  
381 their size and other properties as they move. Hence, Fig. 2d is not so much trajectories, or even a  
382 snapshot of the motion but is a 'collage' of some aspects of the circulation.

383 Beginning with the Hadley circulation, sinking is predicted at the poleward end of the Hadley  
384 cell. The analysis in idea #5 suggests that the midlatitude eddies become unstable and dominant at  
385 latitudes higher than the subtropics (say,  $35^\circ$  latitude). This will define our conceptual boundary

386 between Hadley cell and baroclinic eddy dominated circulations; for compactness the HCBE  
387 boundary. It is no coincidence that this HCBE boundary is not far from the observed location of  
388 the surface subtropical highs in different seasons. The subtropical highs fit nicely with the Hadley  
389 circulation in several ways: winds on the equatorward side of those highs are consistent with  
390 the Hadley circulation, from a boundary layer friction argument those highs would have surface  
391 divergence that can support the sinking by the Hadley circulation (if some mechanism, such as  
392 diabatic cooling allows air parcels to sink to lower potential temperatures).

393 The same boundary layer friction argument fosters rising motion for surface lows so some  
394 mechanism is needed to separate the baroclinic lows from the edge of the Hadley cell. Furthermore,  
395 the baroclinic eddies of higher latitudes extract energy from the zonal flow as well as mix it  
396 vertically, both processes weaken the upper level flow. This argument suggests that the HCBE  
397 boundary is also likely to be a region where the zonal flow is a maximum and that could be a  
398 prototype for the subtropical jet stream.

399 The Hadley cell may extend a bit further poleward in one region due to such things as topo-  
400 graphic deflection of the flow, interaction with higher latitude eddies, and land–sea contrasts.  
401 Angular momentum conservation implies a stronger jet (and presumably greater vertical shear)  
402 in that extended portion of the Hadley cell. Enhanced tropical convection (Sardeshmukh and  
403 Hoskins, 1988) in one region (such as a landmass) may also enhance the subtropical jet in a  
404 region. It is a bit simplistic, but one might argue that such a region would favor the baroclinic  
405 eddy growth compared with a region where the Hadley cell did not extend as far. If one allows this  
406 simplistic argument, then a remarkable chain of events unfolds. The eddies take time to grow,  
407 and they tend to move with that zonal flow when their amplitude is small. This stage of growth is  
408 termed “linear” in the sense that the eddy wind speeds are “small” compared to the mean flow. The  
409 linear stage is also the time when the stability criteria mentioned above have any relevance. While  
410 the eddy may grow by means that alter its shape (primarily the amount of upstream tilt) during  
411 this stage, the orientation and motion of the eddy along the wind flow maximum is sustained.  
412 When the eddy grows to such amplitude that its winds are comparable to the mean flow, then  
413 nonlinear effects become dominant. Nonlinear effects include such things as: enhanced upper  
414 level amplitude, momentum fluxes, and less efficient poleward heat transport (though still large).  
415 Of most interest to the problem of matching circulations is that the baroclinic highs and lows  
416 tend to migrate to the side of the jet-like flow having the same sign of vorticity. The lows migrate  
417 to the poleward side of the jet, while the highs migrate to the equatorward side. The baroclinic  
418 highs can thereby reinforce the subtropical highs while the lows carry precipitation to the middle  
419 latitudes where precipitation has its secondary maxima.

420 The primary function of the circulation is to transport heat poleward and that also needs to be  
421 matched. Developing baroclinic lows and highs at the poleward edge of the Hadley circulation  
422 help to carry further the poleward heat transport by the Hadley cell. Baroclinic highs that migrate  
423 equatorward to reinforce the subtropical highs also assist the heat transport. Lagging behind  
424 (upstream) the surface baroclinic highs are upper level troughs which also assist in picking up the  
425 heat transport by the Hadley cell and carrying it to higher latitudes.

#### 426 4. Summary and conclusions

427 The principal balances and fundamental relationships were described for the large scale  
428 atmospheric circulation. Some implications of these properties were discussed. For example,  
429 a qualitative discussion illustrated how a mountain torque could balance a uniform westerly wind,  
430 though the resultant pressure distribution is opposite to that observed in subtropical latitudes.

431 From this foundation a variety of candidate circulations were proposed and tested against the six  
432 principal balances and observed facts.

433 A hybrid circulation combining a lower latitude “Hadley” cell paired with middle and higher  
434 latitude baroclinic eddies can satisfy the criteria specified. This circulation looks a lot like the  
435 observed circulation, which is no surprise since our proposed circulations were strongly influenced  
436 by foreknowledge of the actual circulation, in particular knowledge of the primary maxima and  
437 minima of precipitation.

438 There are several points to draw from the discussion. First, one is reminded that satisfying  
439 known balances places several constraints upon what circulation is workable. Second, that one  
440 can avoid any mention of a Ferrel cell circulation, instead placing appropriate emphasis upon  
441 baroclinic eddies dominating the midlatitude circulation. Third, that understanding the general  
442 circulation requires looking at fields other than the wind field. Finally, there are many details  
443 that have been glossed over in this discussion. Such “details” can be subtle and challenging and  
444 constitute important subjects of the other articles in this special issue.

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#### 451 **References**

- 452 Baines, P., 1976. The stability of planetary waves on a sphere. *J. Fluid Mech.* 73 (2), 193–213.  
453 Baines, P., Frederiksen, J., 1978. Baroclinic instability on a sphere in two-layer models. *Q. J. R. Met. Soc.* 104, 45–68.  
454 Charney, J., Stern, M., 1962. On the stability of internal baroclinic jets in a rotating atmosphere. *J. Atmos. Sci.* 19, 159–172.  
455 Frederiksen, C., Frederiksen, J., 1990. Flow over topography and instability on a sphere. *J. Atmos. Sci.* 48, 2411–2425.  
456 Grotjahn, R., 1993. *Global Atmospheric Circulations: Observations and Theory*. Oxford, New York, 430 pp.  
457 Grotjahn, R., Castello, C., 2002. Nonmodal growth on a sphere at various horizontal scales. *Geophys. Astrophys. Fluid  
458 Dyn.* 96, 223–238.  
459 Gustafson, W., Weare, B., 2004. MM5 modeling of the Madden-Julian oscillation in the Indian and west Pacific oceans:  
460 model description and control run results. *J. Climate* 17, 1320–1337.  
461 Mak, M., 1983. On moist quasi-geostrophic barotropic instability. *J. Atmos. Sci.* 40, 2349–2367.  
462 Peixoto, J., Oort, A., 1992. *Physics of Climate*. Amer. Int. of Physics, New York, 520 pp.  
463 Phillips, N., 1954. Energy transformations and meridional circulations associated with simple baroclinic waves in a  
464 two-level, quasi-geostrophic model. *Tellus VI*, 273–286.  
465 Sardeshmukh, P., Hoskins, B., 1988. The generation of global rotational flow by steady idealized tropical divergence. *J.  
466 Atmos. Sci.* 45, 1228–1251.