

# Statistical methods for relating temperature extremes to Large-Scale Meteorological Patterns

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**Introduction:** There is a long tradition of using statistical methods based on extreme value theory to estimate the probability of, or return levels for, extreme weather and climate events (Gumbel 1958). Because only a single static probability distribution is used, these methods can be termed unconditional extreme value analysis. In recent years, at least a few papers have made use of statistical methods based on extreme value theory to detect trends in the frequency and intensity of extreme weather and climate events; that is, allowing the extremal distribution to gradually shift over time (Brown et al. 2008).

Only rarely have extreme value distributions for weather and climate extremes been fitted conditional on indices of atmospheric or oceanic circulation, whether climate modes such as the El Niño–Southern Oscillation phenomenon or large-scale meteorological patterns (LSMPs) (Brown et al. 2008). Because the extreme value distribution randomly shifts depending on the value of the circulation index, such methods could be termed conditional extreme value analysis. Using time series of daily maximum temperature for a set of stations in the California Central Valley, we will demonstrate how indices of LSMPs can be incorporated into a conditional extreme value analysis for high temperature extremes.

*Unconditional extreme value analysis:* Classical extreme value analysis involves single (or static) probability distributions (i.e., that do not evolve over time). The Extremal Types Theorem, based on the concept of “max stability,” states that the limiting distribution of the maximum of a sequence of random variables, suitably normalized, is the generalized extreme value (GEV) (Coles 2001). Here, as we shall see, the sequence of random variables need not be temporally independent. Further, conditional extreme value analysis (to be discussed next) provides one avenue for relaxing the assumption that the random variables be identically distributed.

The GEV distribution has three parameters: (i) a location parameter that centers the distribution; (ii) a scale parameter that governs the spread of the distribution; and (iii) a shape parameter that governs how rapidly the upper tail of the distribution decays (by

convention, a negative shape parameter indicates a bounded upper tail as would be anticipated for daily maximum temperature). The block maxima approach to unconditional extreme value analysis entails fitting the GEV distribution to observed maxima (e.g., seasonal highest daily maximum temperature).

An alternative to the block maxima approach, potentially more informative and more powerful, is the peaks over threshold (POT) approach (Coles 2001). This approach entails the statistical modeling of the two most basic components of extremes: (i) the rate of occurrence of an extreme event (e.g., exceeding a high threshold); and (ii) the intensity of a given extreme event (e.g., the excess over a high threshold or by how much it is exceeded). The so-called Law of Small Numbers implies that the frequency of occurrence of sufficiently rare extreme events should have approximately a Poisson distribution, and extreme value theory implies that the excess over a sufficiently high threshold should have approximately a generalized Pareto (GP) distribution (Coles 2001). The Poisson distribution has a single rate parameter that equals both its mean and variance, and the GP distribution has two parameters: (i) a scale parameter that governs the “size” of the excesses; and (ii) a shape parameter with the same interpretation as that of the GEV distribution.

*Conditional extreme value analysis:* The basic idea of conditional extreme value analysis is to allow the extremal distribution to be dynamic; that is, shifting depending on the observed value of an index of a climate mode or of an LSMP. For example, Sillmann et al. (2011) analyzed the lowest winter temperature at grid points in Europe for a reanalysis product and for climate model simulations. The GEV distribution was fitted conditional on the value of an index of North Atlantic atmospheric blocking, with the location and scale parameters varying as functions of the index. In statistical terminology, the conditioning variable (e.g., a blocking index) is called a covariate.

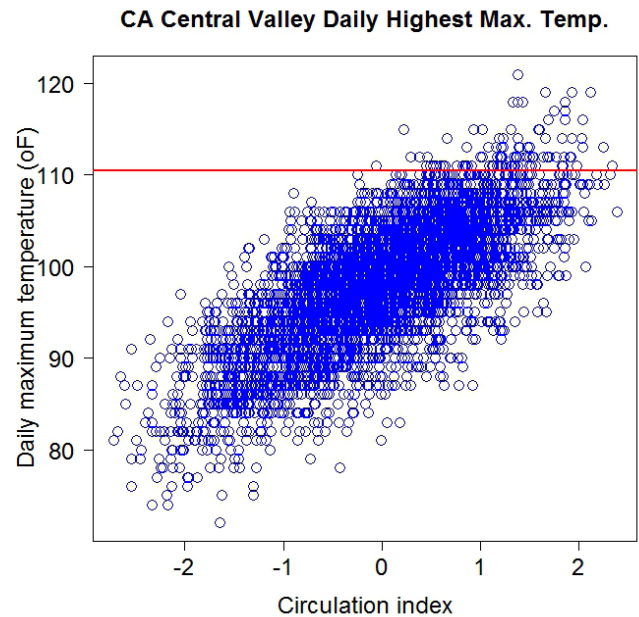
Similarly, the distributions in the POT approach can be conditioned on an LSMP index. In our application to California

temperature extremes, the log-transformed rate parameter of the Poisson distribution and the log-transformed scale parameter of the GP distribution will both be varied as linear functions of an LSMP index (the logarithmic transformation is applied to constrain the parameters to be positive), with the shape parameter of the GP being held constant.

*Application to California temperature extremes:* The climate data consist of time series of daily maximum temperature at three sites, Bakersfield, Fresno, and Red Bluff, in the California Central Valley during the summer season, 16 June to 15 September, over the time period 1951-2005. As a measure of how extreme the temperature is over the entire valley, we extracted the single highest temperature each day. Despite the considerable distances among these three sites, the climatological distributions of summer daily maximum temperatures are quite similar. So the valley-wide temperature extreme is not dominated by a single site.

The LSMP index is discussed in detail in Grotjahn (2011). Briefly, the index is defined by a multi-step process that begins by choosing “target dates.” The target dates satisfy a set of criteria for the extreme event type of interest; here, each date (or first of consecutive dates) is when all three stations spanning the California Central Valley have surface maximum temperature anomaly at least 1.7 standard deviations above normal. These dates happen about 1% of the time. The target dates form an ensemble for each upper air field. Ensemble averages are formed of anomalies of temperature at 850hPa and meridional wind at 700hPa. These ensemble averages are the LSMPs referred to in the present paper and shown in Grotjahn and Faure (2008). The LSMPs have highly significant ridges and troughs spanning the North Pacific and across North America, where significance is determined by bootstrap resampling: comparing the ensemble mean to randomly drawn ensembles at each grid point. Consistency between the ensemble members is assessed using “sign counts” such that only select areas of the ensemble averages (where all members of the ensemble have the same sign) are used to calculate an unnormalized projection. Those select areas sample the significant ridges and troughs of the LSMPs. Finally, the daily LSMP index is a weighted average of those projections of the target ensembles onto each corresponding daily field. The weighting is chosen to optimize when index values exceed a threshold on dates matching target dates over the 1979-1988 training period. Hence, the index estimates how strongly and how similar each day’s upper air fields are to the LSMPs during prior extreme events.

Fig. 1 shows a scatterplot of the highest daily maximum temperature over the California Central Valley versus the LSMP. The horizontal line on the scatterplot indicates a threshold of 110.5 °F to be used in



**Fig. 1** Scatterplot of highest daily maximum temperature over California Central Valley (among Bakersfield, Fresno, and Red Bluff) versus an atmospheric circulation index, 16 June to 15 September, 1951-2005. Horizontal red line indicates threshold of 110.5 °F used in extreme value analysis.

the extreme value analysis. Diagnostics (Coles 2001) indicate that this threshold is sufficiently high to provide an adequate fit of the GP distribution to the temperature excesses. The overall scatterplot suggests a strong relationship between the index and temperature. Nevertheless, our focus is on the points above the threshold for which the nature of the relationship is less clear.

To account for the marked temporal dependence of daily maximum temperature at high levels, the data have been declustered. That is, if the temperature exceeds the high threshold (in our case, 110.5 °F) on two or more consecutive days, only the single highest temperature within the cluster is used. This type of adjustment is termed “runs declustering” with declustering parameter  $r = 1$  (Coles 2001). In other words, each extreme event actually corresponds to a run of consecutive days on which the maximum temperature exceeds the threshold, typically called a hot spell in the climate literature.

For simplicity, the LSMP index for the single day on which the cluster maxima occurs is used as a covariate. All of the statistical analysis was performed using functions in *extRemes*, an open source R package (Gilleland and Katz 2011).

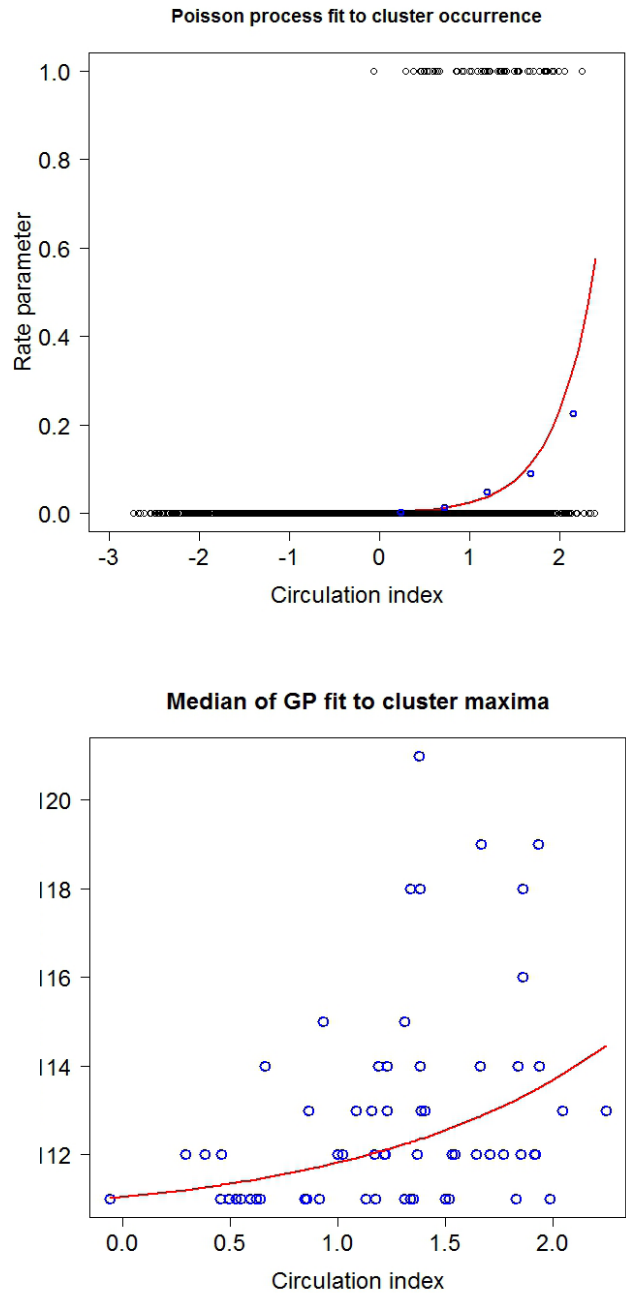
Fig. 2 summarizes the results of applying the POT approach to statistically model the relationship between the highest daily maximum temperature in the California Central Valley and the LSMP index during the summer season.

The top diagram in Fig. 2 shows the fit of a Poisson distribution to the rate of clusters of daily maximum temperature exceeding the threshold, where the log-transformed rate parameter is assumed to be a linear function of the LSMP index. To aid in visualizing the goodness-of-fit, a locally smoothed version of the scatterplot is also included. The fitted statistical model clearly captures the nature of the nonlinearity in the relationship. Further, a likelihood ratio test (Coles 2001), comparing the fit of the statistical model with the LSMP index as a covariate to the fit of a single Poisson distribution, yields a P-value of virtually zero (i.e., overwhelming statistical significance).

The bottom diagram in Fig. 2 shows the fit of a GP distribution to the excess in cluster maxima over the high threshold, where the log-transformed scale parameter is assumed to be a linear function of the LSMP index. For simplicity, only the estimated median of the GP distribution is shown. It rises nonlinearly with the LSMP index, but the high degree of scatter makes it difficult to assess the nature of this relationship. Nevertheless, a likelihood ratio test (Coles 2001), comparing the fit of a GP distribution with the LSMP index as a covariate to the fit of a single GP distribution, yields a P-value of about 0.002 (i.e., strong statistical significance).

It should be noted that the LSMP index was constructed on the basis of the examination of atmospheric circulation patterns on extreme hot days in the Central Valley, using 10 out of the 55 years of data analyzed in the present paper. So these statistical tests of significance should be viewed as a confirmation of the utility of the index, not as an independent analysis.

*Discussion:* Our analysis has focused on high temperature clusters, where the definition of a cluster is based on statistical considerations. Hot spells or heat waves may have more complex definitions that are more meteorologically meaningful (e.g., a heat wave would not necessarily end with the maximum temperature falling below the high threshold for only one day; Meehl and Tebaldi 2004). Further, other characteristics of hot spells or heat waves, including the cluster length, should be statistically modeled as well. Initial attempts to do this include Furrer et al. (2010), which included trend components in a statistical model for hot spells or heat waves, and Photiadou et al. (2014), which included conditioning on indices of atmospheric blocking and climate modes in a similar form of statistical model.



**Fig. 2** Top diagram shows rate parameter of Poisson distribution (red curve) fit to occurrence of cluster of temperatures exceeding threshold of 110.5 °F (black dots: value of “1” indicates occurrence, “0” non-occurrence) with log-linear relationship to atmospheric circulation index and blue dots giving corresponding empirical rate of threshold exceedance. Bottom diagram shows median of GP distribution (red curve) fit to excess in temperature over threshold for cluster maxima (blue dots) with log-linear relationship to circulation index.

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### Acknowledgments

The National Center for Atmospheric Research is sponsored by the National Science Foundation. R. Grotjahn's research is supported by NSF grant 1236681.

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### References

- Brown, S. J., J. Caesar, and C. A. T. Ferro, 2008: Global changes in extreme daily temperature since 1950. *J. Geophys. Res.*, **113**, D05115, doi: 10.1029/2006JD008091.
- Coles, S., 2001: *An Introduction to Statistical Modeling of Extreme Values*. Springer, London, doi: 10.1007/978-1-4471-3675-0.
- Furrer, E. M., R. W. Katz, M. D. Walter, and R. Furrer, 2010: Statistical modeling of hot spells and heat waves. *Climate Res.*, **43**, 191-205, doi: 10.3354/cr00924.
- Gilleland, E., and R. W. Katz, 2011: A new software to analyze how extremes change over time. *Eos*, **92**, 13-14, doi: 10.1029/2011EO020001.
- Grotjahn, R., 2011: Identifying extreme hottest days from large scale upper air data: a pilot scheme to find California Central Valley summertime maximum surface temperatures. *Climate Dyn.*, **37**, 587-604, doi: 10.1007/s00382-011-0999-z.
- Grotjahn, R., and G. Faure, 2008: Composite predictor maps of extraordinary weather events in the Sacramento California region. *Wea. Forecasting*, **23**, 313-335, doi: 10.1175/2007WAF2006055.1.
- Gumbel, E. J., 1958: *Statistics of Extremes*. Columbia University Press, New York.
- Meehl, G., and C. Tebaldi, 2004: More intense, more frequent, and longer lasting heat waves in the 21st century. *Science*, **305**, 994-997, doi: 10.1126/science.1098704.
- Photiadou, C., M. R. Jones, D. Keellings, and C. F. Dewes, 2014: Modeling European hot spells using extreme value analysis. *Climate Res.*, **58**, 193-207, doi: 10.3354/cr01191.
- Sillmann, J., M. Croci-Maspoli, M. Kallache, and R. W. Katz, 2011: Extreme cold winter temperatures in Europe under the influence of North Atlantic atmospheric blocking. *J. Climate*, **24**, 5899-5913, doi: 10.1175/2011JCLI4075.1.

## Self-Organizing Maps: A method for analyzing Large-Scale Meteorological Patterns associated with extreme events

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**I**ntroduction: A climatological goal of extremes analysis is to extract common physical behavior across extreme events so as to gain insight into the causes and maintenance of extreme events. Depending on the phenomenon, such analysis may rely on several fields, such as mean sea-level pressure; mass, energy, moisture and momentum fluxes; and winds, temperature, humidity and geopotential heights at various atmospheric levels. One way of distilling common physical behavior from multiple fields is to form composites across many extreme events at the time of event occurrence and perhaps also at times preceding the events.

However, if there are multiple causes of extremes or if different events occur in different parts of an analysis region, then compositing all events may yield a muddled, and perhaps misleading, picture. Methods that group together events with similar characteristics can avoid this problem and potentially allow more physically relevant composites. One approach to this is Self-Organizing Maps (SOMs).

**SOMs – Overview:** Self-Organizing Maps (SOMs; Kohonen 1995) are two-dimensional arrays of maps that display characteristic behavior patterns of a field (e.g., Cavazos 2000; Hewitson and Crane 2002; Gutowski et al. 2004; Cassano et al. 2007). In comparison with more traditional approaches to investigating multi-dimensional data (e.g. empirical orthogonal functions) the SOM approach compares favorably (Reusch et al. 2005) with distinct advantages in interpreting underlying physical processes. SOMs can reveal observed and simulated evolution of targeted fields, including periodic behavior, provide a basis for estimating statistical significance of climate-change differences, and support conditional compositing of interacting fields and development of probability distributions. Using SOMs, one can assess physical interactions within a model and, further, determine how well a model agrees with observations for sound, physical reasons. SOMs thus give a quantitative, dynamic perspective on climatic behavior and differences between periods and data sources examined.