Winter 2011 ATM 111 Mix of linear & nonlinear Advection

Homework #6

22 pts

1. Understanding how a forecast model predicts a future state. Let the initial condition be:

$U(x, 0) = 0.2*[1-\cos(0.8*x)]^2$ (1) where x ranges from 0 to $2\pi/0.8$.

$U(x, 0) = 0$ (2) where x ranges from $2\pi/0.8$ to 3π

Calculate the model's forecast using 31 grid points at these locations: $x_i = 0$ to 3π in increments of $\pi/10$. U's are nondimensional. Your model is the 1-dimensional *nonlinear* advection equation:

$$U_t + \{c+U\} U_x = 0 \quad (3)$$

Where c = 0.25. Solve (3) subject to these boundary conditions:

$U(0, t) = 0.$ and $U(3\pi, t) = 0.$ (4)

a. (4 pts) Print the values of x and the corresponding U(x, 0) for all 31 grid points.

b. Step (3) forward in time using adjacent cells in a spread sheet program (or using finite differences and looping in a MATLAB or FORTRAN or C++ program). Use the time interval of dt = 0.125. For the first time step use this form:

$$U(x, dt) = U(x, 0) - dt * \{c+U(x, 0)\} * \{U(x+dx, 0) - U(x-dx, 0)\} / (2*dx) \quad (5)$$

For all subsequent time steps use this form:

$$U(x, j*dt) = U(x,(j-2)*dt) - dt*\{c+U(x,(j-1)*dt)\}*\{U[x+dx,(j-1)*dt] - U[x-dx,(j-1)*dt]\} /dx \quad (6)$$

Where j in (6) ranges from 2 to NT and NT is dictated by the length of the integration. Integrate until t=3, so that the number of time steps for the given dt value is 24 (not counting t=0).

b. (6 pts) Print the values of U(x, 1), U(x, 2), and U(x, 3), make sure the x values associated with each U value is indicated unambiguously.

c. (10 pts) Plot the values of U(x, 0), U(x, 1), U(x, 2), and U(x, 3) on ONE chart; make sure your axes are properly labeled (2pt ea curve; 1pt ea axis).

d. (2 pts) Comment on how your solution changes over time.

Notes: The boundary conditions (4) mean you do not calculate U at those locations, though you do need to have those values when doing each time step. You only use formulas (1) and (2) to define the values (e.g. in spreadsheet cells) at the initial time, you do not use that formula at any later time step.

Notes: Please remember that you are to do individual work on all lab exercises. It is OK to use a spread sheet, such as EXCEL, if you wish. Either full or no credit is possible if you simply present a table of numbers; so it is recommended that you show your work as much as possible. Points will be deducted if the data are not labeled, incorrectly labeled, labeled ambiguously, etc. If you use a spreadsheet, email a copy to the Reader when you turn in your assignment.

Assigned: 8 Feb 2011 Due ____ 17 February 2011 _____