Synoptic meteorology Chapter 6.4 & 7.1

March 5 2015
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- ✓ Ads/disads of EBVE
- ✓ Illustration of 500 mb steering

- ✓ Comparing the formula of thermal winds in Holton's & Carlson's
- ✓ Derivation of BEBVE
- ✓ How affect does thermal adv. to local change of perturbation?

Ads/disads of EBVE

$$\frac{\partial \zeta_*}{\partial t} + V_* \cdot \nabla_p (\zeta_* + f) = \frac{\omega_s f_0}{p_s} \overline{A^2(p)} \quad (6.6)$$

A. advantages:

i. Looks formally like the BVE (3.6c) which is easy to understand & interp results of…

ii. Could relate ω to W (ω= dp/dt, hydrostatic eq. ω~-ρgW). This W could come from (a) flow up and down topography, or (b) frictional sfc convergence or divergence

iii. Sign of div correction term (RHS of 6.6) helps slow down long waves. These waves move too fast (retrograde) in the BVE. Related with this to $\frac{\partial Z}{\partial t}$ to see how the term can be brought to the LHS and included in the local change of vort.

B. Limitations:

i. many approx's used: QG assmpt.: vert. variation ignored by taking integral, etc.

ii. "*" level assumed to be fixed in space & time.

iii. A(P) assumed not a function of x, y, and t.

iv. No turning of wind allowed (isotherms and isohypes are paralleled.) ➔ no Temp advection possible.

$$\frac{d\zeta^*}{dt} + \beta v_* = \frac{\omega_s f_0}{p_s}\overline{A^2} = -\frac{f_0 g \overline{A^2}}{p_s} w_s = \frac{f_0 g \overline{A^2}}{p_s}\frac{dz}{dt}$$

$$\frac{f_0 g}{p_s}\overline{A^2} \equiv c > 0, \quad \zeta^* \propto -a^2k^2 z$$

eg $z = -a\sin\{k(x+y)+c\}$

$$\frac{d}{dt}(\zeta^* + cz) = -\beta v_*$$

$$\frac{d}{dt}\left[(-a^2k^2 + c)z\right] = -\beta v_*$$

$$(c - a^2k^2)\left(\frac{\partial z}{\partial t} + \vec{V}_* \cdot \nabla z\right) = -\beta v_*$$

$$\boxed{\frac{\partial z}{\partial t} = -\vec{V}_* \cdot \nabla z + \frac{\beta}{(a^2k^2 - c)} v_*}$$

$$\frac{\partial z}{\partial t} = -\vec{V}_* \cdot \nabla z + \frac{\beta}{a^2k^2} v_*$$ without RHS term (w_s)

$$\frac{1}{(a^2k^2 - c)} \quad \text{vs} \quad \frac{1}{(a^2k^2)}$$

i) if k is large $\frac{1}{(a^2k^2-c)} \sim \frac{1}{a^2k^2}$

no big difference

ii) if k is small (long wave), $(a^2k^2 - c) < 0$.

then, $\frac{\partial z}{\partial t}$ can be attenuated (slow down)

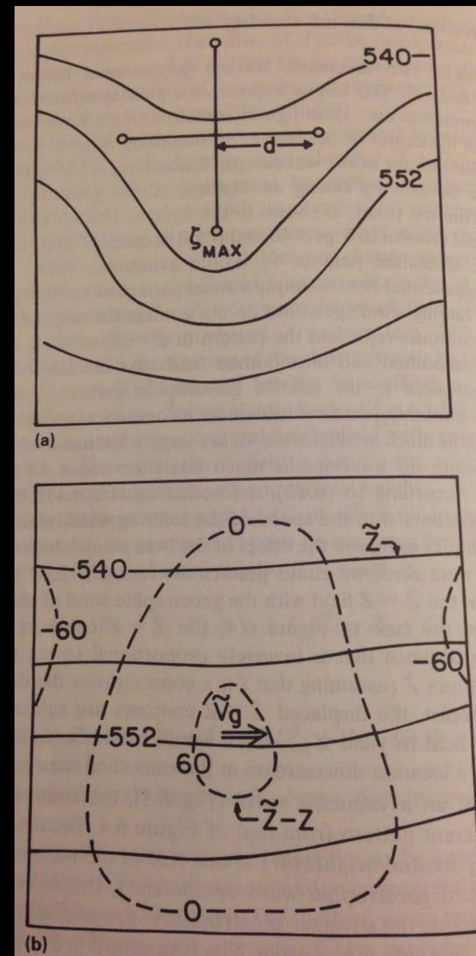
6.4 Illustrations of 500 mb steering

- “*” level is similar to the data on a 500 mb map.

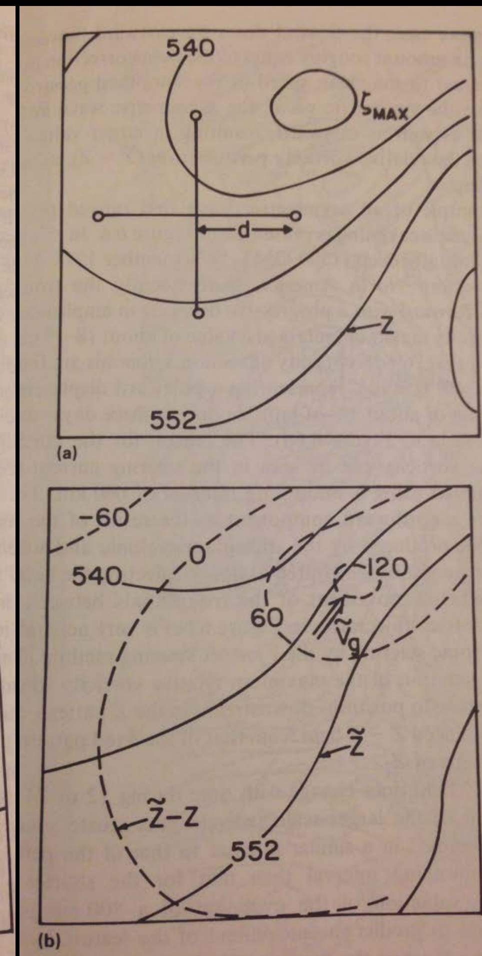
- “linearization”=separating the “perturbation” of trough or ridge from its “environment”. (skipping the linearization step, the Fjortoft scheme described in Chapter 6.3, it is sufficient for us to use the figs 6.4 and 6.5 to illustrate the idea.)

a. Assess the scale of the feature → d=1/4L
b. Apply smoother to total field (=Z) which removes the perturbation (scales similar to 4d & smaller)→Z_tilda
c. Identify a feature to be the “perturbation” (Z_tilda-Z)
d. Feature moves with geostrophic wind of the environment (smoothed wind velocity), Vg_tilda ($k \cdot \nabla \times Z\_tilda$)

In the case of,
1. beta = zero
2. considering beta effect

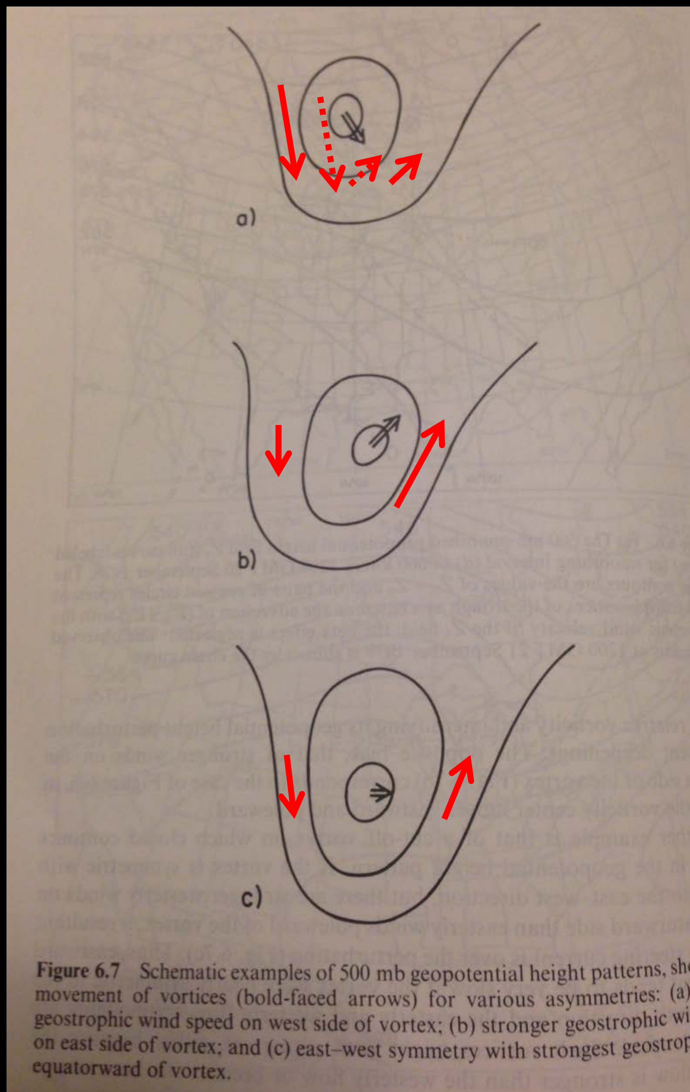


For a symmetric wave



For an asymmetric wave

Asymmetry → the direction (meridional) of system movement



a) Often seen along west coast

b) Often seen in central US

In real world… (Fig 6.6)

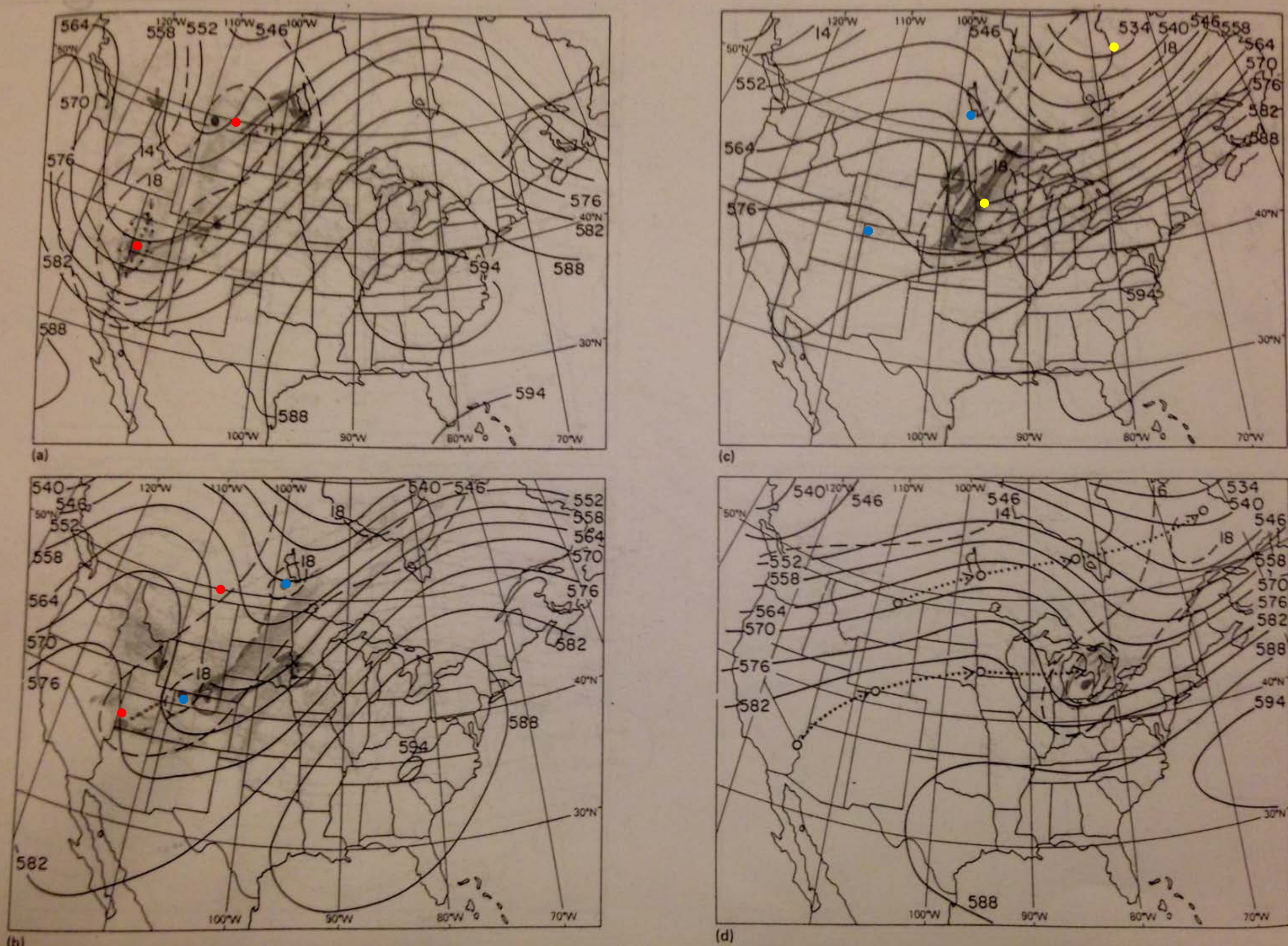


Figure 6.6 (a) The 500 mb geopotential height field (full curves labeled in dam) for 1200 GMT 19 September 1978. Absolute vorticity isopleths (units in $1 \times 10^{-5} s^{-1}$) are drawn as broken curves for values greater than or equal to $14 \times 10^{-5} s^{-1}$. (b) Same as (a) but for for 1200 GMT 20 September 1978.

Figure 6.6 (c) Same as (a) but for for 1200 GMT 21 September 1978. (d) Same as (a) but for for 1200 GMT 22 September 1978. Dotted streamlines with circles denote trajectories of positive vorticity centers at 24 h intervals.

a) rather symmetric

a)→b): red→blue

Rather zonal disp.

b) asymmetric

b)→c): blue→ yellow

large northward disp. of the system

Chapter 7 Baroclinic Development

- Barotropic models (BVE, EBVE): no turning of wind w/ elevation → T advection is missing

- Including the missing process (something similar to T advection) with simple approx.

:

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- Chapter 7.1 : the 2-parameter model (Baroclinie Equivalent Barotropic Vorticity Equation, BEBVE)

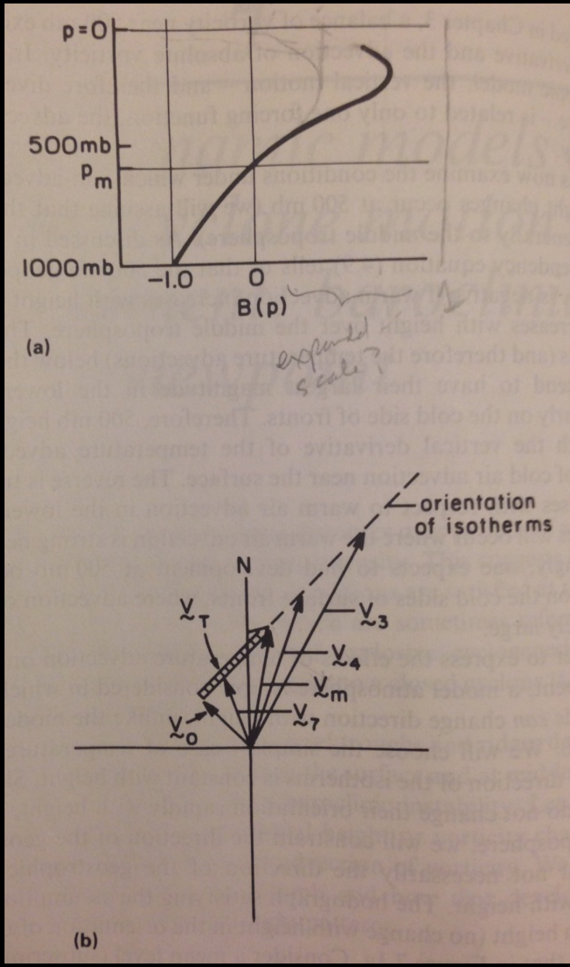
BEBVE

- To allow wind turning simply…
- Assumptions: isotherms are oriented the same direction at all levels. Thermal winds are independent to P. But, magnitude varies with height (controlled by B(p)).
- Mean level, P_m

$$\boldsymbol{V_g = V_{gm} + B(p)V_T} \text{ (7.1a)}$$
$$\boldsymbol{\zeta_g = \zeta_{gm} + B(p)\zeta_T} \text{ (7.1b)}$$

Define (in Fig 7.1a):
Vertical integral of B should be zero.
B(p=1000hPa) = -1.0 at sfc
B(p=0) = 0.0 at upper lid

At sfc, B=-1
$\boldsymbol{V_T = V_m - V_0}$ (Figure 7.1b)



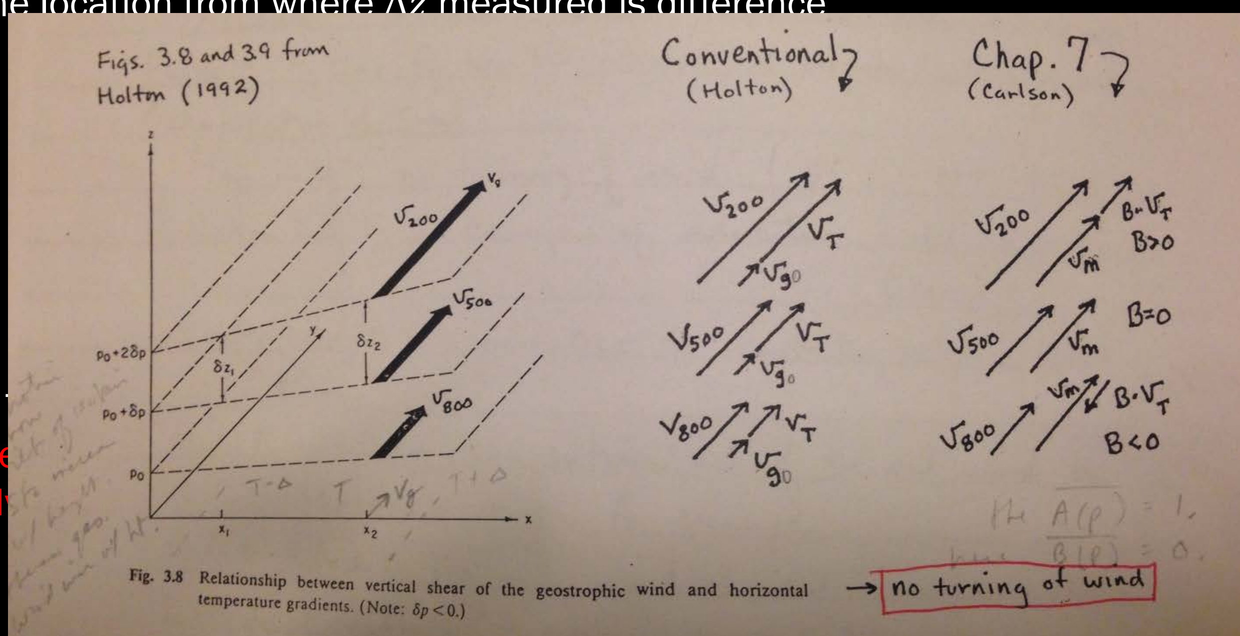
Thermal winds
Conventional (Holton) VS Chap. 7 (Carlson)

$$\boldsymbol{V} = \boldsymbol{V_0} + \boldsymbol{V_T} \qquad \boldsymbol{V} = \boldsymbol{V_m} + \boldsymbol{B(p)V_T} \text{ (7.1a)}$$
$$\boldsymbol{\zeta} = \boldsymbol{\zeta_0} + \boldsymbol{\zeta_T} \qquad \boldsymbol{\zeta} = \boldsymbol{\zeta_m} + \boldsymbol{B(p)\zeta_T} \text{ (7.1b)}$$

- Difference: the reference level (sfc VS mean(middle) level)
  B(p) factor, V_T and $\boldsymbol{\zeta_T}$ are function of p in Holton's, while not in Carlson's.

$$\frac{\Delta V}{\Delta z} \propto -\frac{\Delta T}{\Delta n}$$

Simply the location from where Δz measured is difference



- Case 1:
- Case 2:

Cold adve

Warm ad

Thermal winds

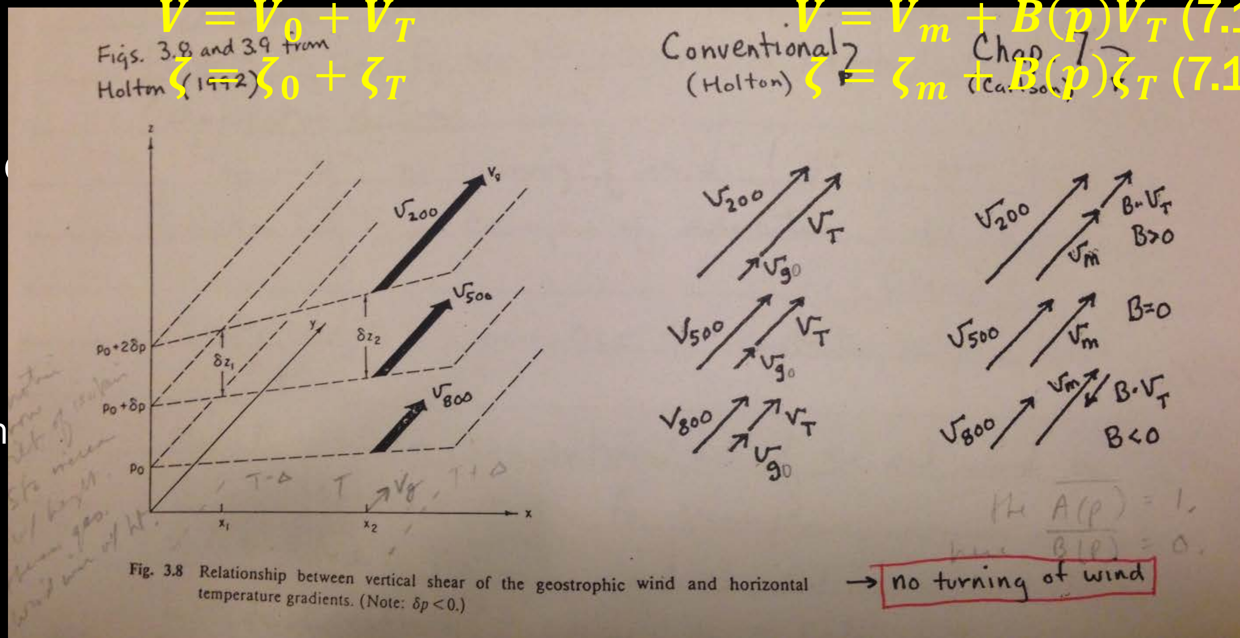
Conventional (Holton) VS Chap. 7 (Carlson)

$V = V_0 + V_T$

$\zeta = \zeta_0 + \zeta_T$

$V = V_m + B(p)V_T$ (7.1a)

$\zeta = \zeta_m + B(p)\zeta_T$ (7.1b)



- Differen… …ot in Carlson's.

Simply th…

- Case 1: No turning of Wind (isohypses and isotherms are paralleled.)

- Case 2: turning case
  Cold advection: backing (CCW)
  Warm advection: veering (CW)

Thermal winds

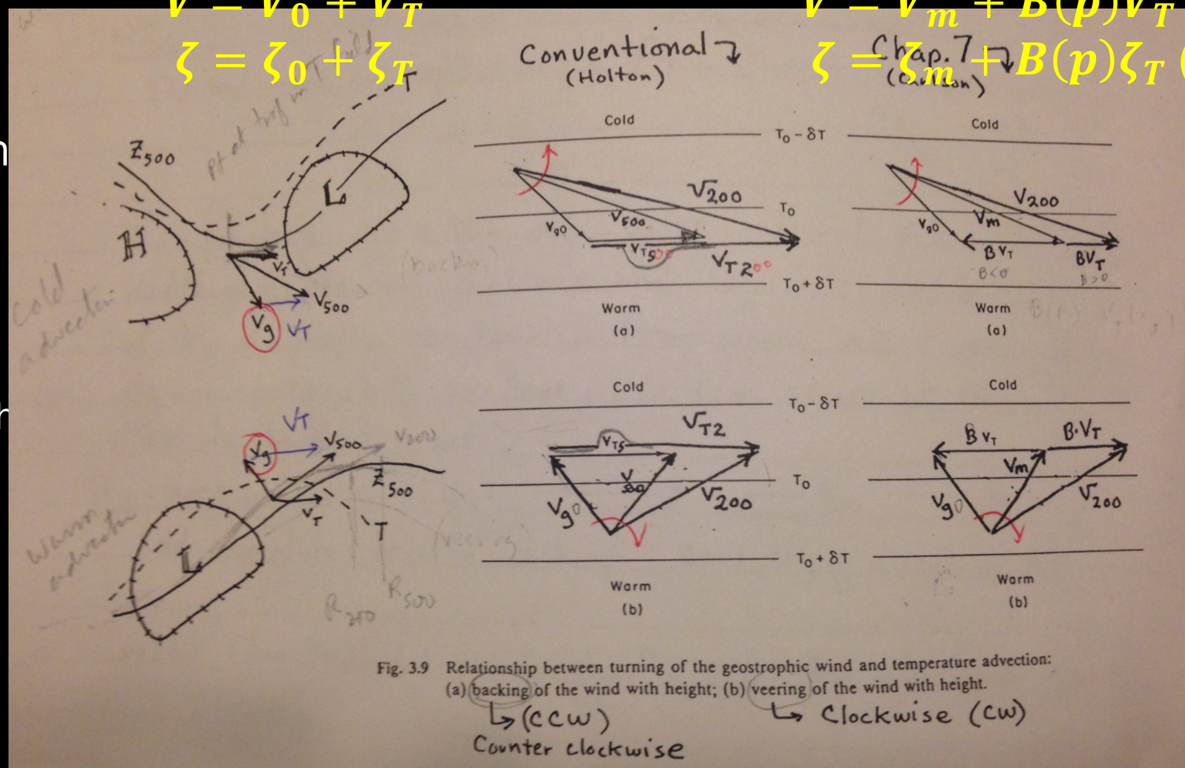
Conventional (Holton) VS Chap. 7 (Carlson)

$$V = V_0 + V_T \qquad V = V_m + B(p)V_T \text{ (7.1a)}$$

$$\zeta = \zeta_0 + \zeta_T \qquad \zeta = \zeta_m + B(p)\zeta_T \text{ (7.1b)}$$

- Differen … e not in Carlson's.

Simply th



- Case 1: … d.)
- Case 2: turning case
  Cold advection: backing (CCW)
  Warm advection: veering (CW)

Derivation of eq. 7.3 (BEBVE)

$$\frac{d(\zeta+f)}{dt} = \frac{\partial \zeta_g}{\partial t} + \boldsymbol{V_g}\cdot\boldsymbol{\nabla_p}(\boldsymbol{\zeta_g}+\boldsymbol{f}) = (\zeta_g+f)\frac{\partial\omega}{\partial p} \approx \boldsymbol{f_0}\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{p}} \quad (3.6a)$$

$$\boldsymbol{V_g} = \boldsymbol{V_{gm}} + \boldsymbol{B(p)V_T} \quad (7.1a)$$
$$\boldsymbol{\zeta_g} = \boldsymbol{\zeta_{gm}} + \boldsymbol{B(p)\zeta_T} \quad (7.1b)$$

1) Sub (7.1a) and (7.1b) into (3.6a)
2) Take pressure average

Tendency term:

$$\frac{1}{P_s}\int_0^{p_s}\frac{\partial\zeta_g}{\partial t}\,dp$$
$$= \frac{1}{P_s}\int_0^{p_s}\left(\frac{\partial\zeta_m}{\partial t} + B\frac{\partial\zeta_T}{\partial t}\right)dp$$
$$= \frac{1}{P_s}\int_0^{p_s}\frac{\partial\zeta_m}{\partial t}\,dp + \frac{1}{P_s}\int_0^{p_s}B\frac{\partial\zeta_T}{\partial t}\,dp$$
$$= \frac{\partial\zeta_m}{\partial t}\left(\frac{1}{P_s}\int_0^{p_s}1\,dp\right) + \frac{\partial\zeta_T}{\partial t}\left(\frac{1}{P_s}\int_0^{p_s}B\,dp\right) = \frac{\partial\zeta_m}{\partial t}$$

Since, $\frac{\partial\zeta_m}{\partial t}$ and $\frac{\partial\zeta_T}{\partial t}$ are independent of pressure, $\frac{1}{P_s}\int_0^{p_s}1\,dp$=1, and $\frac{1}{P_s}\int_0^{p_s}B\,dp$=0

Nonlinear term:

$$\frac{1}{P_s}\int_0^{p_s}\Big((\boldsymbol{V_m}+\boldsymbol{BV_T})\cdot\boldsymbol{\nabla_p}(\boldsymbol{\zeta_m}+\boldsymbol{B\zeta_T}+\boldsymbol{f})\Big)dp$$
$$= \frac{1}{P_s}\int_0^{p_s}\big(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p\zeta_m} + \boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p\zeta_m} + \boldsymbol{V_m}\cdot\boldsymbol{\nabla_p}(\boldsymbol{B\zeta_T}) + \boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p}(\boldsymbol{B\zeta_T})$$
$$+ \boldsymbol{V_m}\cdot\boldsymbol{\nabla_p f} + \boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p f}\big)\,dp$$
$$= \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p\zeta_m})\,dp + \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p\zeta_m})\,dp$$
$$+ \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p}(\boldsymbol{B\zeta_T}))\,dp$$
$$+ \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p}(\boldsymbol{B\zeta_T}))\,dp$$
$$+ \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p f})\,dp + \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{BV_T}\cdot\boldsymbol{\nabla_p f})\,dp$$
$$= \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p\zeta_m})\,dp + (\boldsymbol{V_T}\cdot\boldsymbol{\nabla_p\zeta_T})\frac{1}{P_s}\int_0^{p_s}B^2\,dp$$
$$+ \frac{1}{P_s}\int_0^{p_s}(\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p f})\,dp$$
$$= [\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p}(\boldsymbol{\zeta_m}+\boldsymbol{f})]\cdot\left(\frac{1}{P_s}\int_0^{p_s}1\,dp\right)$$
$$+ \overline{\boldsymbol{B^2}}(\boldsymbol{V_T}\cdot\boldsymbol{\nabla_p\zeta_T})$$
$$= \boldsymbol{V_m}\cdot\boldsymbol{\nabla_p}(\boldsymbol{\zeta_m}+\boldsymbol{f}) + \overline{\boldsymbol{B^2}}(\boldsymbol{V_T}\cdot\boldsymbol{\nabla_p\zeta_T})$$

Vertical term: $\frac{1}{P_s}\int_0^{p_s}\left(\boldsymbol{f_0}\frac{\partial\boldsymbol{\omega}}{\partial\boldsymbol{p}}\right)dp = \frac{f_0}{P_s}\int_0^{p_s}\partial\boldsymbol{\omega} = \frac{f_0}{P_s}(\omega_s-\omega_0) = \frac{f_0\omega_s}{P_s}$

$$\frac{\partial\boldsymbol{\zeta_m}}{\partial\boldsymbol{t}} = -\boldsymbol{V_m}\cdot\boldsymbol{\nabla_p}(\boldsymbol{\zeta_m}+\boldsymbol{f}) - \overline{\boldsymbol{B^2}}(\boldsymbol{V_T}\cdot\boldsymbol{\nabla_p\zeta_T}) + \frac{f_0\omega_s}{P_s} \quad (7.3)$$

$$\frac{\partial \zeta_m}{\partial t} = -\boldsymbol{V}_m \cdot \boldsymbol{\nabla}_p(\zeta_m + f) - \overline{B^2}(\boldsymbol{V}_T \cdot \boldsymbol{\nabla}_p \zeta_T) + \frac{f_0 \omega_s}{P_s}$$

- Similar to (6.6), $\frac{\partial \zeta_*}{\partial t} = -\boldsymbol{V}_* \cdot \boldsymbol{\nabla}_p(\zeta_* + f) + \frac{f_0 \omega_s}{P_s} \overline{A^2(p)}$
- The new term is a possible source or sink of mean vorticity, "development" by Carlson.

- How does the development term affect the tendency of Zeta_m?

- $\zeta_T \propto \nabla^2 h \propto -h \propto -T$ (from hypsometric eq.)
  then, $\frac{\partial \zeta_m}{\partial t} \sim \overline{B^2}(-V_T \cdot \nabla_p \zeta_T) \sim -\overline{B^2}(-V_T \cdot \nabla_p T)$
  ➔ eventually, we check an analogy of temp. adv. is included in BEBVE.

- Cold advection $-V_T \cdot \nabla_p T < 0$ means POS vor. Adv. $-V_T \cdot \nabla_p \zeta_T > 0$, then Zeta_m increases.
- Warm advection $-V_T \cdot \nabla_p T > 0$ means NEG vor. Adv. $-V_T \cdot \nabla_p \zeta_T < 0$, then Zeta_m decreases.

$$\frac{\partial \zeta_m}{\partial t} = -V_m \cdot \nabla_p(\zeta_m + f) + \overline{B^2}(-V_T \cdot \nabla_p \zeta_T) + \frac{f_0 \omega_s}{P_s}$$

$V_g = V_m + B(p)V_T$ (7.1a)

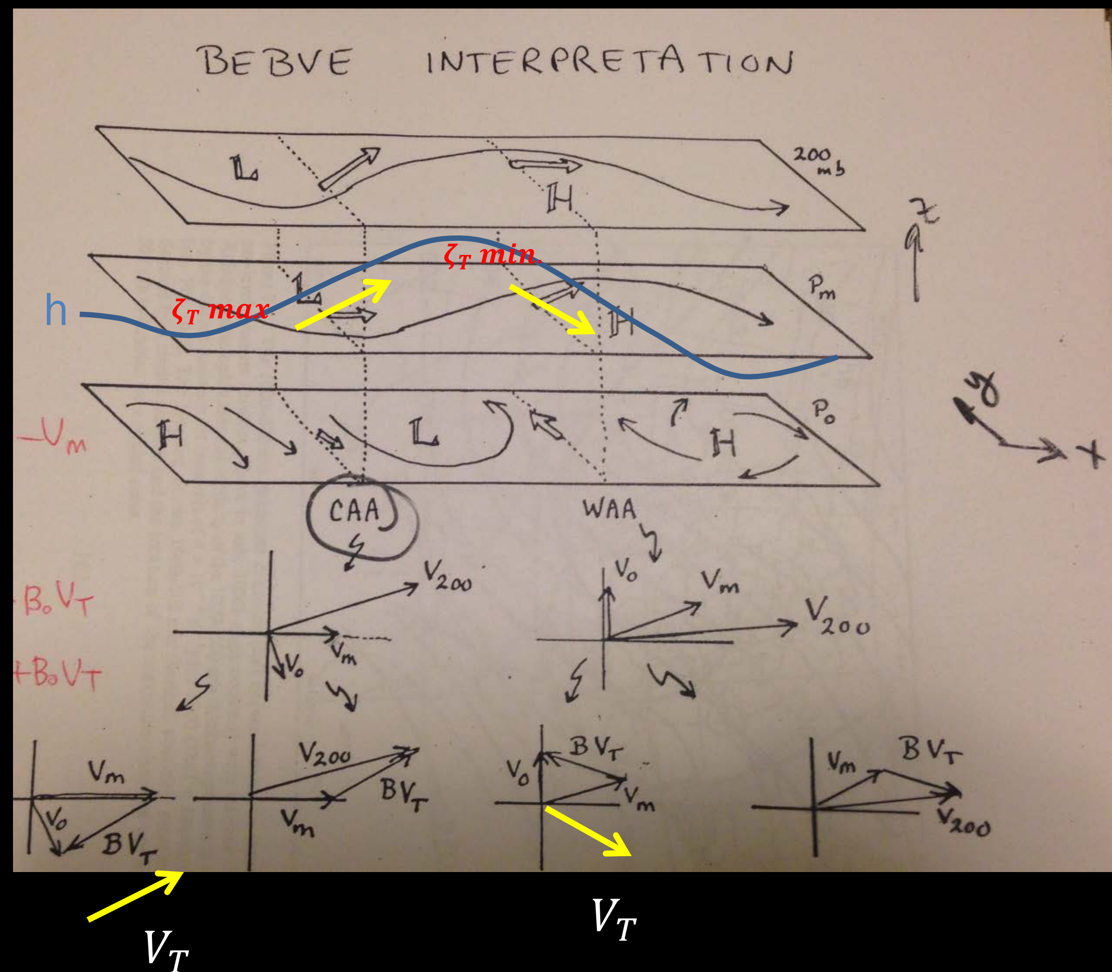
- Example 1

$$B(p)V_T = V_g - V_m$$

$$B(0)V_T = V_0 - V_m$$
$$B(200)V_T = V_{200} - V_m$$

At sfc, B=-1
$V_T = V_m - V_0$

- $\zeta_T \propto \nabla^2 h \propto -h \propto -T$



$$\frac{\partial \zeta_m}{\partial t} = -V_m \cdot \nabla_p(\zeta_m + f) + \overline{B^2}(-V_T \cdot \nabla_p \zeta_T) + \frac{f_0 \omega_s}{P_s}$$

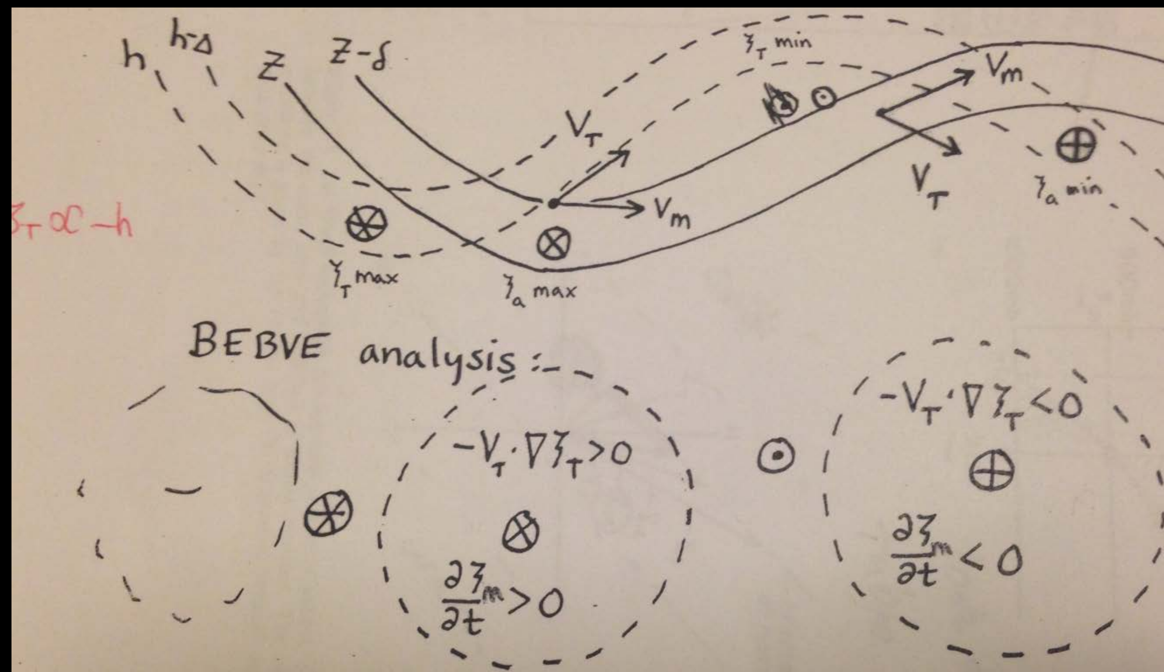
- Example 2

"x" circle: min. z → max. abs. vor.
"*" circle: min. h → max. thermal vor.
"+" circle: max. z → min. abs. vor.
"." circle: max. h → min. thermal vor.

- V_T ≈isotherms
- V_m ≈isobars

- There are dipole patterns of tendency at a tough & a ridge
- h and Z patterns are offset which is a necessary condition for the system "developing".

Q. what if the patters are offset in opposite way?



$$\frac{\partial \zeta_m}{\partial t} = -\boldsymbol{V_m} \cdot \nabla_p(\zeta_m + f) + \overline{B^2}\left(-\boldsymbol{V_T} \cdot \nabla_p \zeta_T\right) + \frac{f_0 \omega_s}{P_s}$$

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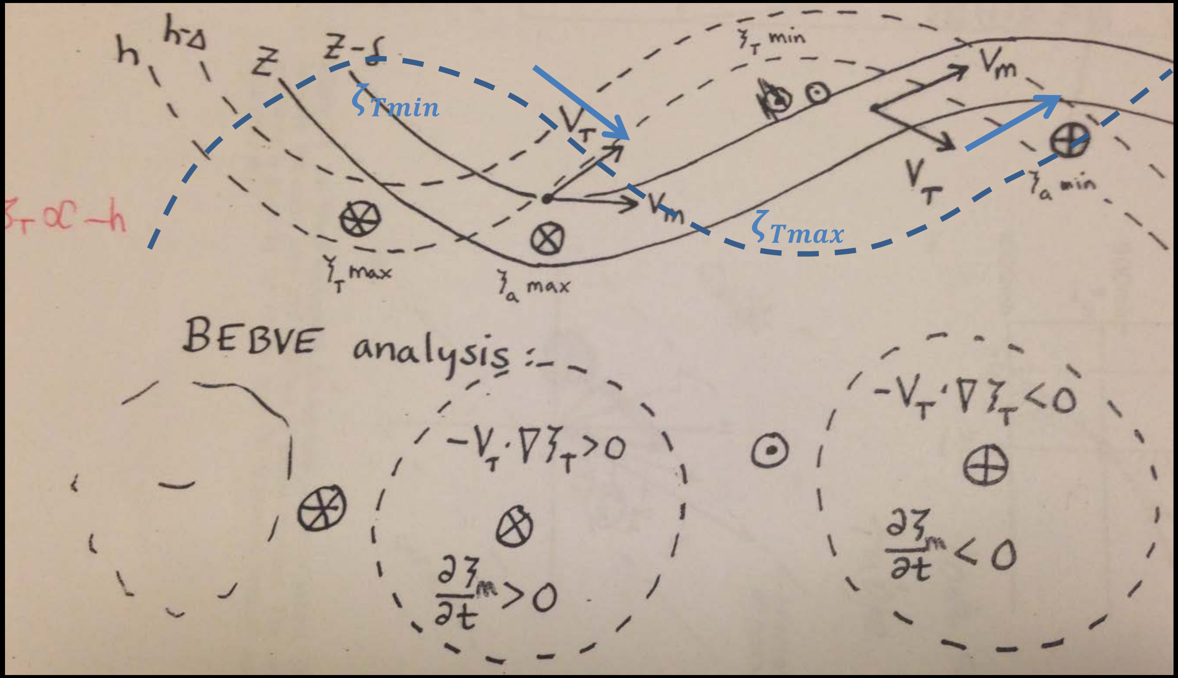
- There are dipole patterns of tendency at a tough & a ridge
- h and Z patterns are offset which is a necessary condition for the system "developing"

Q. what if the patters are offset in opposite way? For the system "decaying"



$-\boldsymbol{V_T} \cdot \boldsymbol{\nabla_p} \zeta_T < 0$

$\frac{\partial \zeta_m}{\partial t} < 0$

$-\boldsymbol{V_T} \cdot \boldsymbol{\nabla_p} \zeta_T > 0$

$\frac{\partial \zeta_m}{\partial t} > 0$

$$\frac{\partial \zeta_m}{\partial t} = -\boldsymbol{V}_m \cdot \nabla_p(\zeta_m + f) + \overline{B^2}\left(-\boldsymbol{V}_T \cdot \nabla_p \zeta_T\right) + \frac{f_0 \omega_s}{P_s}$$

- Figure 7.2
- Solid: 1000-500 thickness
- Dashed: SLP
- Dotted: thermal vorticity

- "*" circle: max. thermal vor.
- "x" circle: max. abs. vor. (trough) at 500 mb

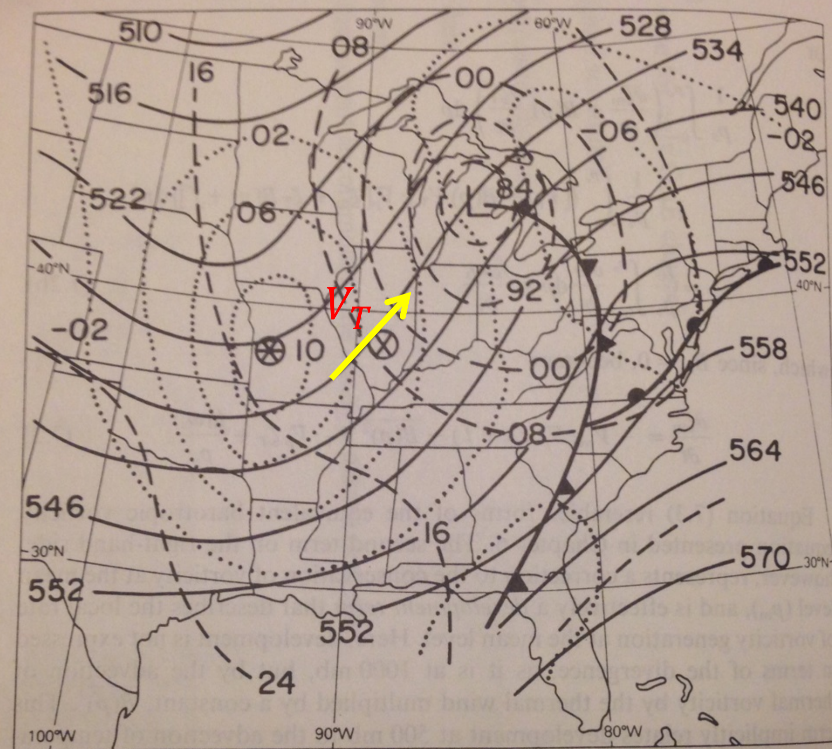


Figure 7.2 The 1000–500 mb thickness contours (full curves labeled in dam) and sea-level pressure (broken curves in mb; 1000s digit omitted) with surface fronts. Relative geostrophic thermal vorticity of the 1000–500 mb thickness pattern is shown by the dotted contours (at intervals of $4 \times 10^{-5}\ s^{-1}$) for 1200 GMT 4 January 1982 (see Fig. 10.5). The location of the 1000–500 mb thermal vorticity maximum is denoted by a circled asterisk and the location of the maximum absolute vorticity at 500 mb is denoted by a circled cross.

Applications

- To apply directly to standard weather maps….
- Approximate form of (7.3) based on Carlson's experience,

$$\frac{\partial \zeta_m}{\partial t} = -\boldsymbol{V}_m \cdot \boldsymbol{\nabla}_p(\zeta_m + f) + \overline{B^2}\left(-\boldsymbol{V}_T \cdot \boldsymbol{\nabla}_p \zeta_T\right) + \frac{f_0 \omega_s}{P_s} \quad (7.3)$$

$\overline{B^2(p)} = 0.2$ is a good estimator **IF** applying at the 500 mb level and for smooth terrain ($\omega_s \sim 0$).
Table 6.1

From (7.3), vorticity tendencies at 500 mb,

$$\frac{\partial \zeta_{g5}}{\partial t} \approx -V_{g5} \cdot \nabla_p\left(\zeta_{g5} + f\right) - 0.2\left(V_T \cdot \nabla_p \zeta_T\right) \quad (7.4a)$$

$$\frac{\partial \zeta_{g5}}{\partial t} \approx -V_{g5} \cdot \nabla_p\left(\zeta_{g5} + f\right) = \left(\frac{\partial \zeta_{g5}}{\partial t}\right)_{barotropic} \quad (3.6c)$$

$$\frac{\partial \zeta_{g5}}{\partial t} \approx \left(\frac{\partial \zeta_{g5}}{\partial t}\right)_{barotropic} - 0.2\left(V_T \cdot \nabla_p \zeta_T\right)$$

From BVE(3.6c) and since $\zeta \propto \nabla^2 Z \propto -Z,\ \zeta_T \propto \nabla^2 h \propto -h$,

$$\frac{\partial Z_5}{\partial t} \approx \left(\frac{\partial Z_5}{\partial t}\right)_{barotropic} - 0.2\left(V_T \cdot \nabla_p h\right)$$

Empirical eq. using h=1000-500mb thickness, V_{1000} instead of V_T,

$$\boxed{\frac{\partial Z_5}{\partial t} \approx \left(\frac{\partial Z_5}{\partial t}\right)_{barotropic} - 0.2\left(\boldsymbol{V}_{g0} \cdot \boldsymbol{\nabla}_p \boldsymbol{h}\right) \quad (7.4b)}$$

Applications

- To apply directly to standard weather ma
- Approximate form of (7.3) based on Carls

$$\frac{\partial \zeta_m}{\partial t} = -\boldsymbol{V}_m \cdot \boldsymbol{\nabla}_p(\zeta_m + f) + \overline{B^2}(-\boldsymbol{V}_T \cdot \boldsymbol{\nabla}_p \zeta_T)$$

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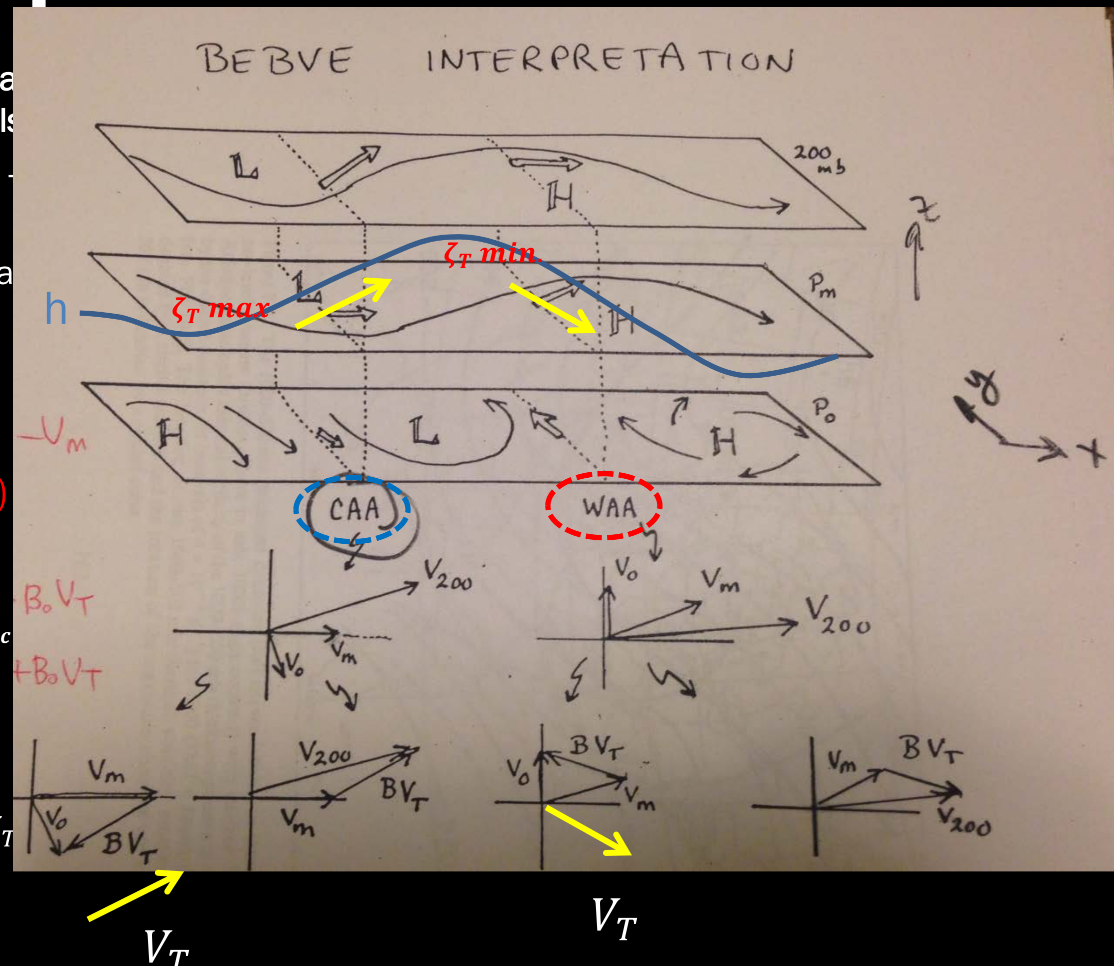

From (7.3), vorticity tendencies at 500 mb,

$$\frac{\partial \zeta_{g5}}{\partial t} \approx -V_{g5} \cdot \nabla_p(\zeta_{g5} + f) - 0.2(V_T \cdot \nabla_p \zeta_T)$$

$$\frac{\partial \zeta_{g5}}{\partial t} \approx -V_{g5} \cdot \nabla_p(\zeta_{g5} + f) = \left(\frac{\partial \zeta_{g5}}{\partial t}\right)_{barotropic}$$

$$\frac{\partial \zeta_{g5}}{\partial t} \approx \left(\frac{\partial \zeta_{g5}}{\partial t}\right)_{barotropic} - 0.2(V_T \cdot \nabla_p \zeta_T)$$

From BVE(3.6c) and since $\zeta \propto \nabla^2 Z \propto -Z,\ \zeta_T$

$$\frac{\partial Z_5}{\partial t} \approx \left(\frac{\partial Z_5}{\partial t}\right)_{barotropic} - 0.2(V_T \cdot \nabla_p h)$$

Empirical eq. using h=1000-500mb thickness, V_{1000} instead of V_T,

$$\frac{\partial Z_5}{\partial t} \approx \left(\frac{\partial Z_5}{\partial t}\right)_{barotropic} - 0.2(\boldsymbol{V}_{g0} \cdot \boldsymbol{\nabla}_p h) \quad (7.4b)$$

Summary

- ✓ Ads/disads of EBVE
  - → no temp. adv. Possible
- ✓ Illustration of 500 mb steering
  - → Asymmetry & the direction of system movement
- ✓ Comparing the formula of thermal winds in Holton's & Carlson's
  - → Looking at the same features despites of different formula
- ✓ Derivation of BEBVE
  - → Includes temp. adv. with turning winds in height
- ✓ How affect does thermal adv. to local change of perturbation?
  - → More accurate estimation of local change of perturbation possible from thermal advection
  - → Prediction from the snap shot of thermal adv, yay!