

Comprehensive Final Exam — 2014
65 pts possible

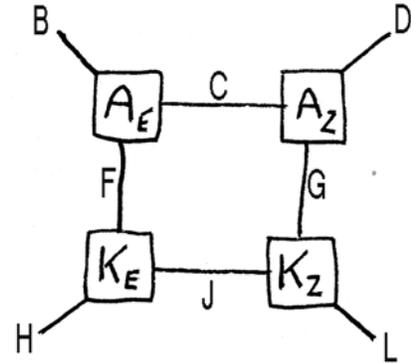
1. (10 pts) Consider the energy box figure **on this page**. Use the letter labels provided.

a. (3 pts) Draw arrows to indicate the direction of energy flow for these quantities: D, F, and H.

b. (3 pts) Match these letters in the figure at left with the attached choices drawn from equations (4.62) through (4.69)

J in figure is eqn: _____

B in figure is eqn: _____ C in figure is eqn: _____



c. (3 pts) Fill in the correct letter. The energy conversion

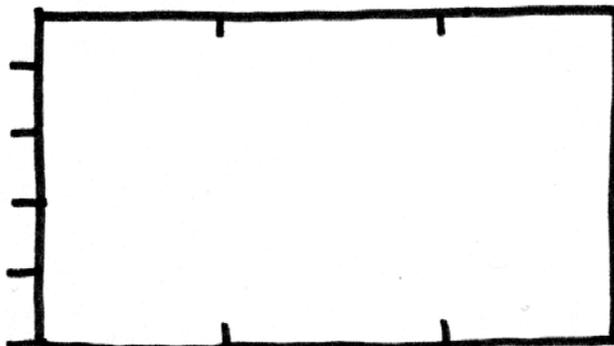
_____ is estimated as a residual in practice. In contrast,

the energy conversion _____, is estimated by an approximate formula that includes parts $[u_g]$ and $[v]$.

The 'barotropic' conversion is letter _____.

d. Describe in a simple sentence how available potential energy can be created.

2. (4 pts) Momentum flux. Draw a clearly labeled diagram showing at least 3 contours of 500 hPa geopotential height in a latitude versus longitude plot with a contour pattern that implies **geostrophic winds** producing a **northward** flux of zonal average eddy momentum. Indicate representative latitude and longitude values at the tick marks as well as contour values with a 100m interval. Finally, make your region located in the **Southern** hemisphere.



3. (3 pts) Write down the formula for the zonal average of T at 30 degrees north and 200 mb. Define all variables you introduce.

4. (5 pts) True (T) or false (F), continued.

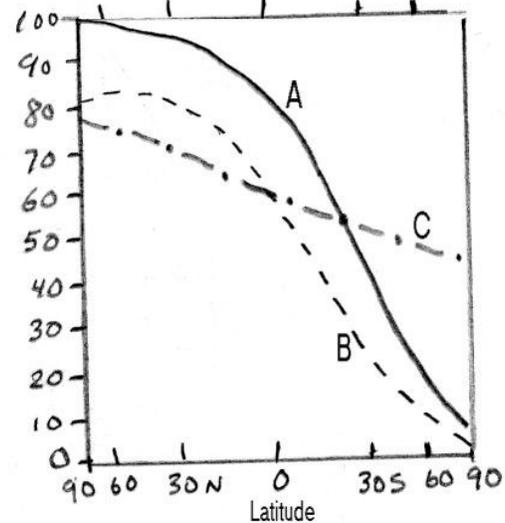
- T F Western boundary currents are found along the west coasts of some continents.
- T F At pressure levels below 500 hPa, the Hadley cell transports moist static energy equatorward
- T F The efficiency factor, ϵ , is mainly positive in the middle troposphere of the tropics.
- T F The *range of latitudes* having surface westerlies is greater than the range of the latitudes having surface easterlies.
- T F The rising branch of the Hadley cell mainly occurs in thunderstorms.

5. (6 pts) Radiative balance for Uranus. Uranus has a tilt of the rotation axis of > 80 degrees such that the north pole faces the sun far more than the south pole. A colleague at NASA asks you to help her interpret the radiation data plotted on the chart at right by answering the questions:

a. (1.5 pts) Which curve is what? (Place the letter in the blank)

- ___ incoming solar radiation
- ___ outgoing IR radiation
- ___ absorbed solar radiation

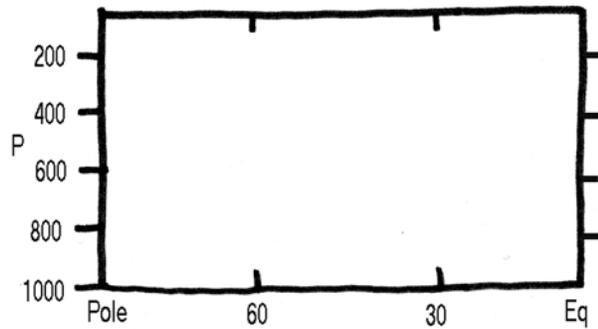
- b. The albedo at the north pole is _____ percent.
- c. Net radiation is zero at latitude _____ degrees
- d. The heat transport is a maximum at latitude _____ degrees
- e. (1.5 pts) There is something clearly wrong with the data.
Explain why something in this data cannot be correct.



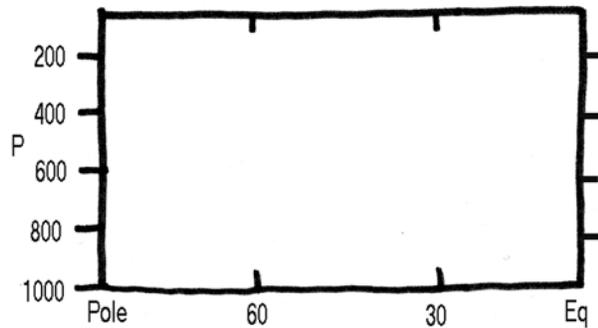
6. The Kuo-Eliassen equation.

a. (8 pts) Explain how a Ferrel cell maintains thermal wind balance in the presence of eddy fluxes. (Do not consider diabatic processes). Include discussion of the eddy and Ferrel cell properties.

b. (2 pts) On the panel at right, draw a schematic diagram of the meridional cross section of the observed eddy meridional potential temperature heat fluxes: $[\Theta'v']$. Be sure to define the convention you use to represent the fluxes, including what is positive versus negative.



c. (3 pts) Let the Kuo-Eliassen equation be approximated by: $-\Psi = -M^2[\Theta'v']/My^2$. Using your answer in part b., deduce the streamfunction field (Ψ). Draw the Ψ field on the chart at right being consistent with what you drew above. Be sure to indicate **clearly the sign(s)** in your answer.



7. (5 pts) Angular momentum. a) Let M be angular momentum per unit mass for an air parcel moving at zonal speed S for Jupiter rotating with angular rate A , having radius r , at latitude L , for air density d , and air pressure P . Using these letters, write down the formula for M .

$M =$

b. Jupiter takes 3.564×10^4 s to complete 1 revolution. The radius is $r = 6.99 \times 10^7$ m. Find the angular rate, A to 3 significant digits.

8. (11 pts) The **net** latent heat flux across 10N is -10^{15} W. Assume that the meridional velocity field is specified by $v = A*(0.5 - P/P_0)$ and the specific humidity field by: $q = Q (P/P_0)^3$ where A is 2 m/s and Q is to be determined in gm of water vapor per kg of air. P is the pressure elevation which ranges from 0 to P_0 where $P_0 = 10^5$ Pa. Some helpful information is on the attached sheet.
- a. (6 pts) Find Q such that the specified latent heat flux is obtained.

- b. (5 pts) Find the annual rainfall for the area 0 to 10 N assuming that: all the flux across 10 N falls out as rain and is spread evenly over this area. Express your answer in cm/yr. (Hint: first obtain the rain rate, expressed as m of water depth per second.)

9. (8 pts) Perform/answer the specific task/question for the indicated figure.

a. Fig. (1) In the Southern Hemisphere moisture transport is a maximum *northward* at latitude _____ S.

b. Fig. (2) What is the most likely variable plotted here? _____

c. Fig. (3) Based on this plot of $[u'v']$, place an X in each hemisphere (NH and SH) location where the kinetic energy generation by this term is likely to be largest. Be precise in centering in both latitude and pressure since this term has weighting by a function of latitude. 2

d. Fig. (4) Are the highs and lows shown here amplifying or decaying? _____

e. Fig. (5) What 3-month season is most likely shown here? _____.

f. Fig. (5) Place an 'H' inside each Hadley cell and an 'F' inside each Ferrel cell shown in the figure. 2

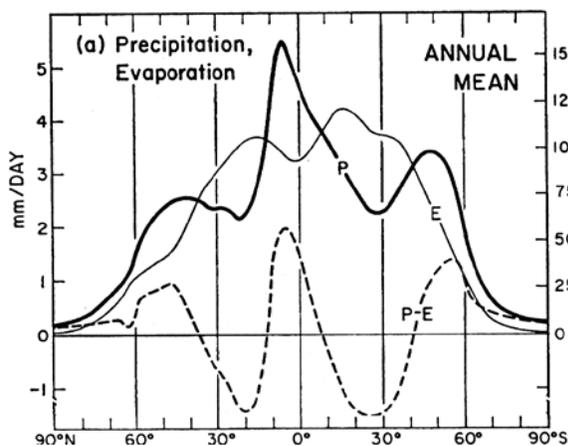


Fig.1

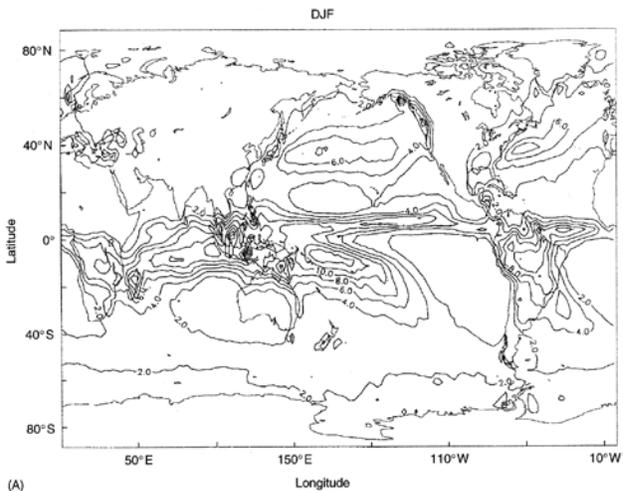


Fig. 2 (above)

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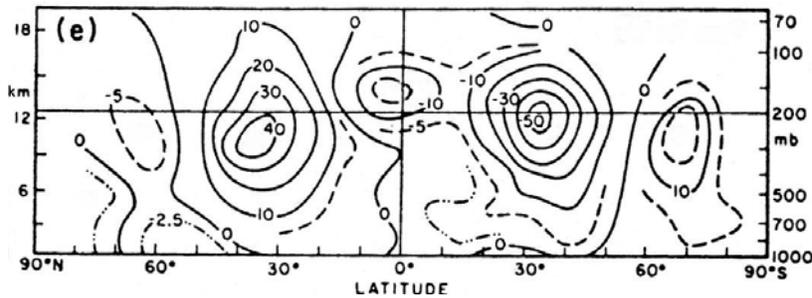


Fig. 3

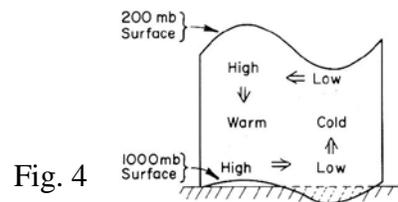
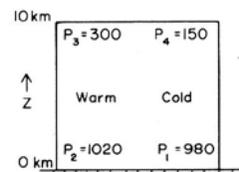
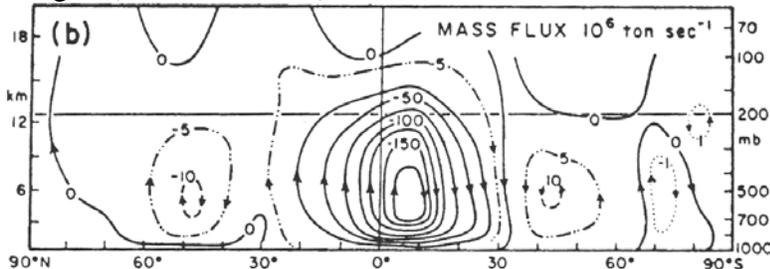


Fig. 4

(a)

Fig. 5 (stream function)



(b)

Some helpful information:

$\sin(0) = 0.$	$\cos(0) = 1$
$\sin(10) = 0.17365$	$\cos(10) = 0.98481$
$\sin(30) = 0.5$	$\cos(30) = 0.86603$
$\sin(38) = 0.61566$	$\cos(38) = 0.78801$
$\sin(90) = 1.$	$\cos(90) = 0$

density of water: $\rho_w = 10^3 \text{ kg/m}^3$

latent heat of vaporization at 20C: $L = 2.4 \times 10^6 \text{ J/kg}$

acceleration of gravity: $g = 9.81 \text{ m/s}^2$

radius of earth: $r = 6.37 \times 10^6 \text{ m}$

gas constant: $R = 287 \text{ J/(K kg)}$

specific heat at constant pressure: $C_p = 1004 \text{ J/(K kg)}$

$$\kappa = R/C_p = 0.28585657$$

angular velocity of rotation: $\Omega = 7.292 \times 10^{-5} \text{ s}^{-1}$

$$\pi = 3.14159$$

conversions:

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ J} = 1 \text{ N-m}$$

$$1 \text{ N} = \text{kg m/s}^2$$

$$1 \text{ Pa} = \text{N/m}^2 = \text{J/m}^3$$

useful formulas:

$$I_i = \frac{C_1}{\lambda^5 [\exp(\frac{C_2}{\lambda T}) - 1]} \quad (3.1)$$

$$\text{heat transport} = \frac{2\pi a \cos \phi}{g} \int [v(C_p T + \Phi + Lq)] dP \quad (3.3)$$

$$\psi = \frac{2\pi R}{g} \int_P^{P_0} [v] dP \quad (3.4)$$

$$[v] = \frac{g}{2\pi R} \frac{\partial \psi}{\partial p} \quad \text{and} \quad [\omega] = \frac{-g}{2\pi r^2 \cos \phi} \frac{\partial \psi}{\partial \phi} \quad (3.5)$$

$$\text{MSE} = \Phi + C_p T + Lq \quad (3.6)$$

$$pz = RT \quad \text{or} \quad p = \rho RT \quad (1.17)$$

$$dp/dz = -\rho g \quad (1.18)$$

$$\mathbf{V}_g \equiv \mathbf{k} \times \frac{1}{\rho f} \nabla p \quad (2.23)$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_P + \frac{\partial \omega}{\partial p} = 0 \quad (3.5)$$

$$\frac{\partial A_Z}{\partial t} = GZ - CZ - CA \quad A_Z = C_p \int (\epsilon |\bar{T}|) dM \quad (4.58)$$

$$\frac{\partial A_E}{\partial t} = GE - CE + CA \quad (4.57) \quad A_E = \frac{C_p}{2} \int \gamma (\bar{T}^*)^2 + (\bar{T}^{**})^2 dM \quad (4.59)$$

$$\frac{\partial K_Z}{\partial t} = -DZ + CZ - CK \quad K_Z = \frac{1}{2} \int (\bar{v})^2 + (\bar{v}^*)^2 dM \quad (4.60)$$

$$\frac{\partial K_E}{\partial t} = -DE + CE + CK \quad K_E = \frac{1}{2} \int [(\bar{u}^*)^2 + (\bar{v}^*)^2] + [(\bar{u}^{**})^2 + (\bar{v}^{**})^2] dM \quad (4.61)$$

where

$$\gamma = - \left(\frac{\theta}{T} \right) \left(\frac{P_r}{P} \right)^{\kappa} \left(\frac{\kappa}{P_r} \frac{\partial P_r}{\partial \theta} \right)$$

$$GZ = \int (\epsilon |\bar{Q}|) dM \quad (4.62)$$

$$GE = \int \gamma (\bar{Q}^* \bar{T}^*) dM \quad (4.63)$$

$$CZ = - \int (\bar{\omega})_p (\bar{\omega})_p dM \quad (4.64)$$

$$CE = - \int (\bar{\omega}^* \bar{\omega}^* + \bar{\omega}^{**} \bar{\omega}^{**}) dM \quad (4.65)$$

$$CK = - \int (\bar{u}^* \bar{v}^* + \bar{u}^{**} \bar{v}^{**}) \frac{\cos \phi}{r} \frac{\partial}{\partial \phi} \left(\frac{\bar{u}}{\cos \phi} \right) dM \quad (4.66)$$

$$- \int (\bar{u}^* \bar{\omega}^* + \bar{u}^{**} \bar{\omega}^{**}) \frac{\partial}{\partial P} (\bar{u}) dM$$

$$CA = - C_p \int \left(\frac{\theta}{T} \right) (\bar{v}^* \bar{T}^* + \bar{v}^{**} \bar{T}^{**}) \frac{1}{r} \frac{\partial}{\partial \phi} \left(\frac{|\epsilon| \bar{T}^*}{|\theta|} \right) dM \quad (4.67)$$

$$- C_p \int \left(\frac{\theta}{T} \right) (\bar{\omega}^* \bar{T}^* + \bar{\omega}^{**} \bar{T}^{**}) \frac{\partial}{\partial P} \left(\frac{|\epsilon| \bar{T}^*}{|\theta|} \right) dM$$

$$DZ = - \int (\bar{u}) (\bar{F}_x) + (\bar{v}) (\bar{F}_y) dM \quad (4.68)$$

$$DE = - \int (\bar{u}^* \bar{F}_x^* + \bar{v}^* \bar{F}_y^* + \bar{u}^{**} \bar{F}_x^{**} + \bar{v}^{**} \bar{F}_y^{**}) dM \quad (4.69)$$

where r is the earth's radius, $dM = r^2 (\cos \phi / g) d\phi d\lambda dP$, λ is longitude, ϕ latitude.

$$\frac{\partial}{\partial t} \underbrace{\int \left(\frac{u^2}{2} \right) dm}_{(A)} + \underbrace{\int \left(\frac{u}{R} \right) \left(\frac{1}{R} \frac{\partial R^2 \{u\omega\}}{\partial y} + \frac{\partial R \{u\omega\}}{\partial P} \right) dm}_{(B)} \underbrace{= 0}_{(C)} \quad (4.12)$$

$$- \int f \{u\} \{v\} dm + \int \{u\} \{F_x\} dm = 0$$

$$\frac{\partial A_z}{\partial t} = \underbrace{\frac{1}{g} \int \int_S \int_{P_2}^{P_1} \{\epsilon q\} dP dS}_{(A)} + \underbrace{\frac{1}{g} \int \int_S \int_{P_2}^{P_1} \{\epsilon \omega \alpha\} dP dS}_{(B)}$$

$$- \underbrace{\frac{C_p}{g} \int \int_S \int_{P_2}^{P_1} \left\{ \nabla_P \cdot (T \nabla_P) + \frac{\partial}{\partial P} (\omega T) \right\} dP dS}_{(C)} \quad (4.31)$$

$$\{v\} = \frac{\partial \psi}{\partial p} \quad \text{and} \quad \{\omega\} = - \frac{\partial \psi}{\partial y} \quad (4.46)$$

$$\frac{\partial \{v_g\}}{\partial p} = \frac{R p^{\kappa-1}}{f P_0^{\kappa}} \frac{\partial \{\theta\}}{\partial y} \equiv \gamma \frac{\partial \{\theta\}}{\partial y} \quad (4.49)$$

$$A \frac{\partial^2 \psi}{\partial y^2} + 2B \frac{\partial^2 \psi}{\partial y \partial p} + C \frac{\partial^2 \psi}{\partial p^2} + D \frac{\partial \psi}{\partial y} + E \frac{\partial \psi}{\partial p} = \gamma \frac{\partial H}{\partial y} + \frac{\partial \chi}{\partial p} \quad (6.50)$$

$$A = -\gamma \frac{\partial \{\theta\}}{\partial p} \quad B = \frac{\partial \{u\}}{\partial p} = \gamma \frac{\partial \{\theta\}}{\partial y} \quad C = f - \frac{\partial \{u\}}{\partial y}$$

$$E = - \frac{\partial \gamma}{\partial y} \frac{\partial \{\theta\}}{\partial y} \quad D = \frac{\partial \gamma}{\partial p} \frac{\partial \{\theta\}}{\partial y}$$

$$\chi = \{F_x\} + \frac{\partial \{u'v'\}}{\partial y} + \frac{\partial \{u'\omega'\}}{\partial p} \quad H = - \left(\frac{\partial \{\theta'v'\}}{\partial y} + \frac{\partial \{u'\omega'\}}{\partial p} \right) + \frac{[Q][\theta]}{[T]}$$

$$\frac{\partial \zeta_a}{\partial t} + \mathbf{V}_\psi \cdot \nabla \zeta_a = -\mathbf{V}_\chi \cdot \nabla \zeta_a - \zeta_a D \equiv S$$