

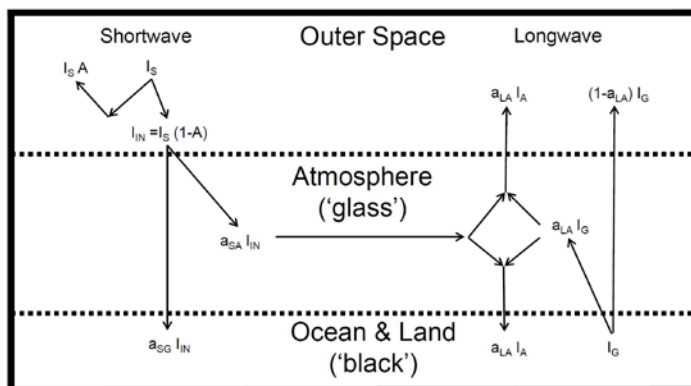
Comprehensive Final Exam — 2016
65 pts possible

1. (16 pts) The glass slab (radiation) model

a. (3 pts) Using the notation in the figure, write down the equations for energy balance for:

The atmosphere:

The surface ('Ocean & Land'):



b. (6 pts) Derive an expression relating I_A to I_{IN} that does not include I_G .

c. (1 pt) What is required of a_{SA} and a_{LA} for I_G to be greater than I_{IN} ? (Hint: if you cannot recall the condition, derive the expression relating I_G to I_{IN} that does not include I_A using the back of this page.

d. (3 pts) Draw a dashed line to indicate how surface latent heat flux (SLHF) be added to this diagram.

When SLHF is positive, how does I_G change?

When SLHF is positive, how does I_A change?

e. (1 pt) Consider a variation on the model having two atmospheric layers with observed atmospheric values. I_1 is from the top layer and I_2 is from the bottom layer, which is greater?

f. (2 pts) Write the formula for the temperature (T) of the atmosphere using the notation of the figure. Define in words any quantities you introduce.

2. (10 pts) Perform/answer the specific task/question for the indicated figure. (Ratios are OK)

a. Fig. (1) Circle one: the season is most likely: DJF, MAM, JJA, SON, ANN (annual ave)

b. Fig. (2) Circle one: the season is most likely: DJF, MAM, JJA, SON, ANN

c. Fig. (2) Draw a solid line on the figure indicating the tropopause.

d. Fig. (3) The albedo of the earth's surface is: _____

e. Fig. (3) The emission to space by the atmosphere (including clouds) is: _____

f. Fig. (3) The incoming solar radiation absorbed by atmosphere and earth is: _____

g. Fig. (3) The absorptivity of the atmosphere to longwave radiation is: _____

h. Figs. (4) or (5) Circle one: the dashed lines are for DJF, MAM, JJA, SON, ANN

i. Figs. (4) or (5) Using the thin solid line, the southward total heat flux is greatest at latitude _____

j. Figs. (4) or (5) Circle one: the time of year the Earth is closest to the sun is: DJF, JJA.

Fig. 1

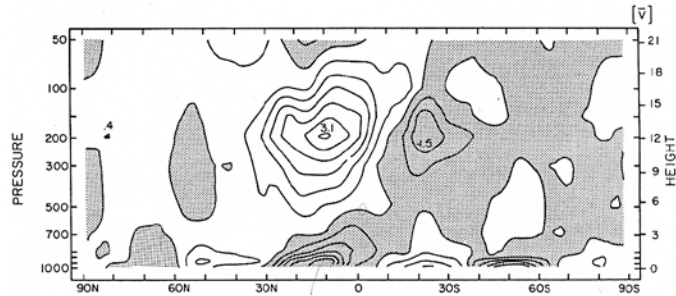


Fig. 2

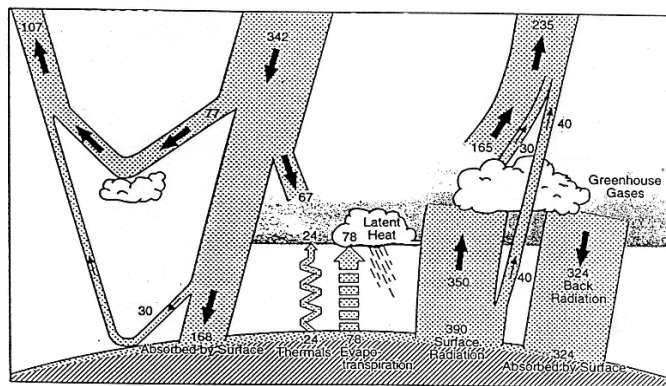
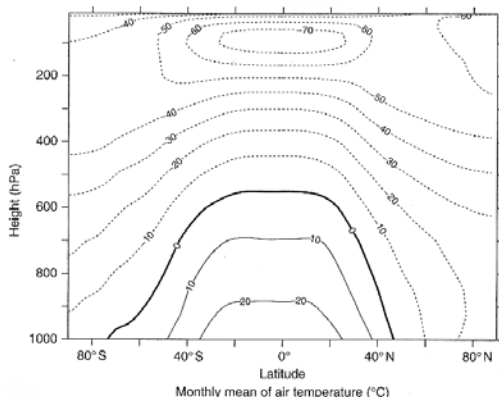


Fig. 3

Fig. 4

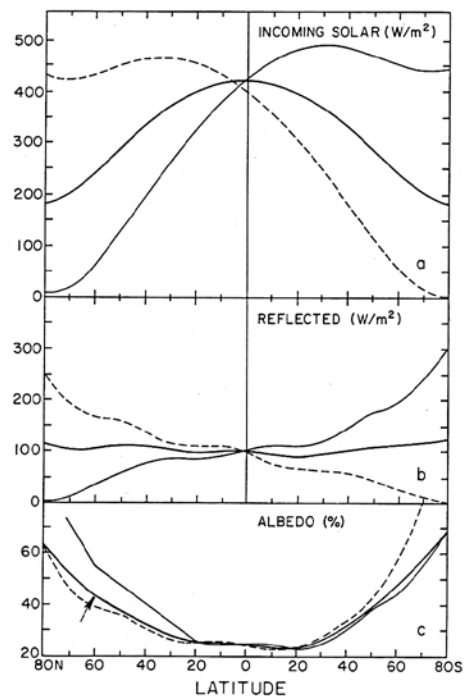
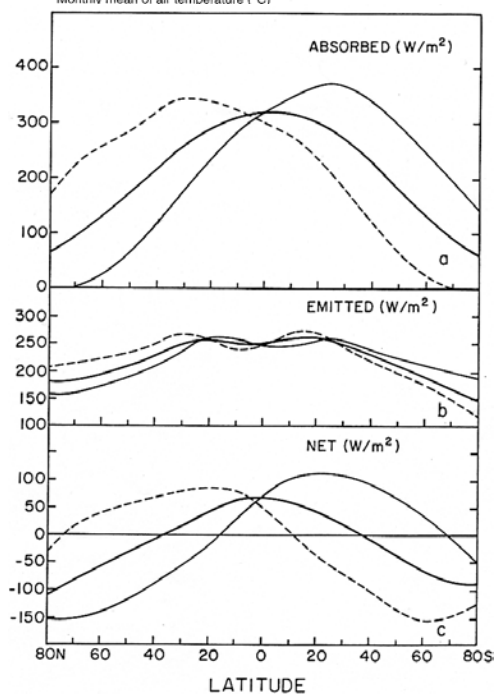


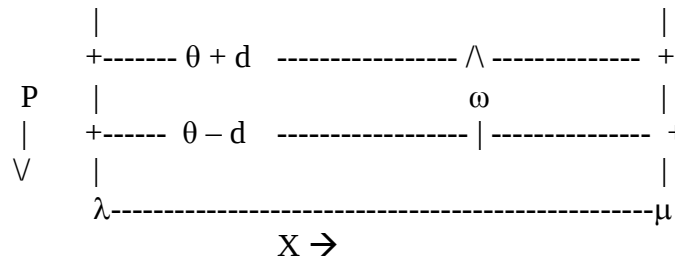
Fig 5

3. (4 pts) Available potential energy (APE). In the figure below, parcels on the right side of the domain are rising adiabatically. Assume that prior to the vertical motion the efficiency factor was zero everywhere in the domain plotted.

a. Deduce the signs of the following quantities when averaged over X. They have the standard meanings of pressure velocity, efficiency factor, specific volume, potential temperature, etc.. The air in the right *half* of the domain is rising (no rising or sinking elsewhere in the domain).

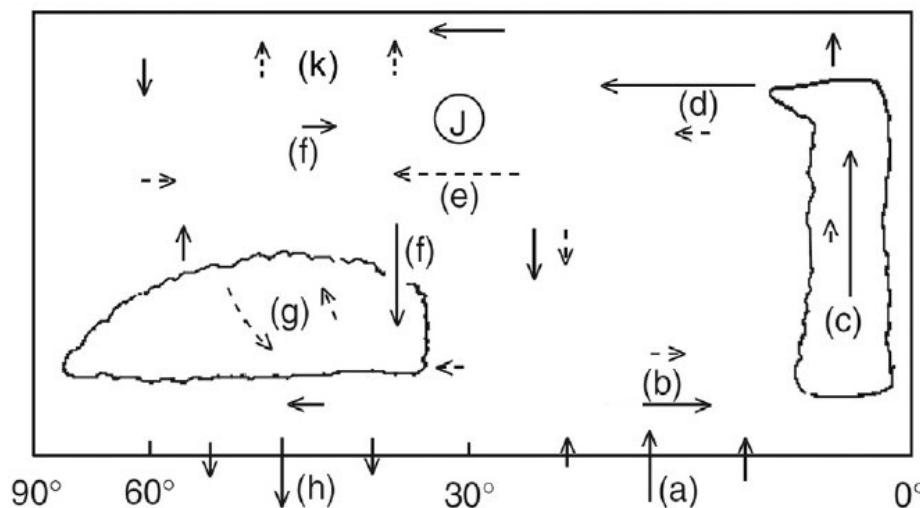
[ω] _____ than zero

[ε] _____ than zero



b. Deduce the sign of $d[\text{APE}]/dt$ from the information given. State your reasoning.

4. (4 pts) Summary chart of momentum cycle in the atmosphere was presented on the course web page and is duplicated below.



Write the correct letter to the left of each process that best matches the letter of the process on the diagram.

- _____ sink of westerly momentum
- _____ source of westerly momentum
- _____ ICZ convective transport of momentum
- _____ upper Ferrel cell momentum flux

5. (2 pts) Using the notation in the Appendix, what types of atmospheric phenomena are included in the momentum flux quantity below:

$$[(u'')^*] [(v'')^*]$$

6. Net energy and mass transports by the Hadley cell. Write down the key quantity that the indicated transport is proportional to in each part. Define all variables. Hint: use of the notation from the Appendix in the Grotjahn (1993) textbook is expected.

a. (4 pts) Meridional mass transport is proportional to:
where:

The *net* meridional mass transport is zero because:

b. (5 pts) Meridional latent heat transport is proportional to:
where:

The *net latent* heat transport is equatorward because:

c. (5 pts) Total atmospheric heat transport is proportional to:
where:

The *net total* atmospheric heat transport is poleward because:

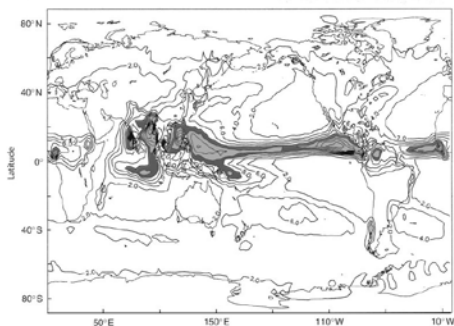
7. (8 pts) True (T) or false (F), circle the correct letter. 1 point each.

- | | | |
|---|---|--|
| T | F | Annual average net radiation is positive over most of the Sahara. |
| T | F | The A_Z to K_Z conversion is proportional to $[v_g][u]$ where the subscript indicates the geostrophic value. |
| T | F | The A_Z to A_E conversion is proportional to $[v' T']$ |
| T | F | The K_E to K_Z conversion is called the barotropic conversion |
| T | F | The A_Z to A_E to K_E conversion is the main route of energy flow within the atmosphere. |
| T | F | In general, eddy available potential energy (A_E) will be created by latent heat release if the condensation is occurring in the warm air sector of a frontal cyclone. |
| T | F | A trough tilted from southwest to northeast has northward momentum flux in the Northern Hemisphere and southward flux in the Southern Hemisphere. |
| T | F | Cold air rising or warm air sinking converts APE into KE. |

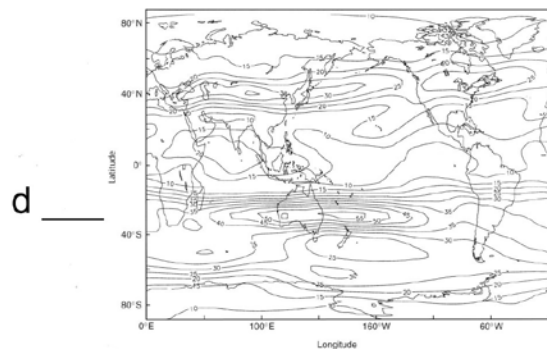
8. (7 pts) Match the description by placing the NUMBER label on the nearest blank line for each plot.

For extra credit, the 3-month season for fig. a is: _____ and the 3-month for fig. e is: _____

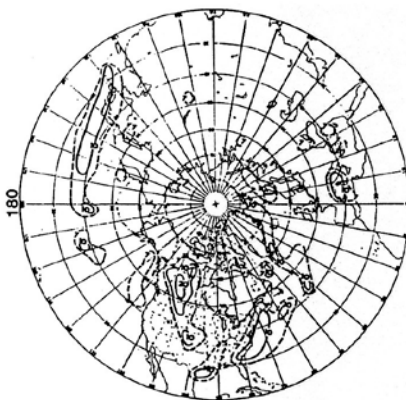
- | | |
|--|---|
| 1. Outgoing longwave radiation | 2. Net radiation |
| 3. Sea level pressure | 4. 700 mb temperature |
| 5. Albedo | 6. Precipitation |
| 7. 850 mb Transient eddy momentum flux | 8. 850 mb Stationary eddy momentum flux |
| 9. Cyclogenesis regions | 10. Cyclolysis (cyclone decay) regions |
| 11. 200 mb eddy momentum flux | 12. 200 mb ageostrophic meridional wind |
| 13. 200 mb wind speed | 14. Absorbed solar radiation |



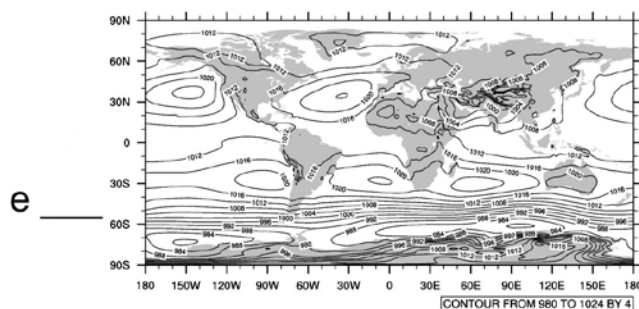
a



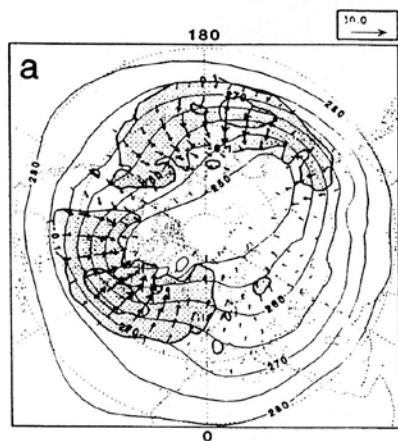
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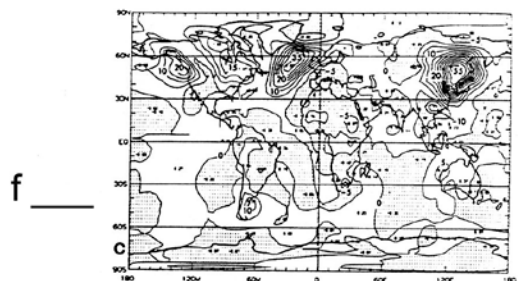
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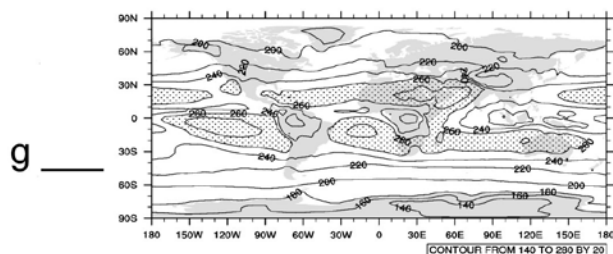
e



c



f



g

Some helpful information:

$\sin(0) = 0.$	$\cos(0) = 1$
$\sin(10) = 0.17365$	$\cos(10) = 0.98481$
$\sin(30) = 0.5$	$\cos(30) = 0.86603$
$\sin(38) = 0.61566$	$\cos(38) = 0.78801$
$\sin(90) = 1.$	$\cos(90) = 0$

density of water: $\rho_w = 10^3 \text{ kg/m}^3$

latent heat of vaporization at 20C: $L = 2.4 \times 10^6 \text{ J/kg}$

acceleration of gravity: $g = 9.81 \text{ m/s}^2$

radius of earth: $r = 6.37 \times 10^6 \text{ m}$

gas constant: $R = 287 \text{ J/(K kg)}$

specific heat at constant pressure: $C_p = 1004 \text{ J/(K kg)}$

$$\kappa = R/C_p = 0.28585657$$

angular velocity of rotation: $\Omega = 7.292 \times 10^{-5} \text{ s}^{-1}$

$$\pi = 3.14159$$

conversions:

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ J} = 1 \text{ N-m}$$

$$1 \text{ N} = \text{kg m/s}^2$$

$$1 \text{ Pa} = \text{N/m}^2 = \text{J/m}^3$$

useful formulas:

$$I_i = \frac{C_1}{\lambda^5 [\exp(\frac{C_2}{\lambda T}) - 1]} \quad (3.1)$$

$$\text{heat transport} = \frac{2\pi a \cos \phi}{g} \int \overline{[v(C_p T + \Phi + Lq)]} dP \quad (3.3)$$

$$\psi = \frac{2\pi R}{g} \int_P^{P_0} [v] dP \quad (3.4)$$

$$[v] = \frac{g}{2\pi R} \frac{\partial \psi}{\partial p} \quad \text{and} \quad [\omega] = \frac{-g}{2\pi r^2 \cos \phi} \frac{\partial \psi}{\partial \phi} \quad (3.5)$$

$$\text{MSE} = \Phi + C_p T + Lq \quad (3.6)$$

$$p\alpha = RT \quad \text{or} \quad p = \rho RT \quad (1.17)$$

$$dp/dz = -\rho g \quad (1.18)$$

$$\mathbf{V}_g \equiv \mathbf{k} \times \frac{1}{\rho f} \nabla p \quad (2.23)$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{\partial \omega}{\partial p} = 0 \quad (3.5)$$

$$\frac{\partial A_Z}{\partial t} = GZ - CZ - CA \quad A_Z = C_p \int [\epsilon] [\bar{T}] dM \quad (4.58)$$

$$\frac{\partial A_E}{\partial t} = GE - CE + CA \quad A_E = \frac{C_p}{2} \int \gamma [(\bar{T}')^2 + (\bar{T}'')^2] dM \quad (4.59)$$

$$\frac{\partial K_Z}{\partial t} = -DZ + CZ - CK \quad K_Z = \frac{1}{2} \int [\bar{v}]^2 + [\bar{v}']^2 dM \quad (4.60)$$

$$\frac{\partial K_E}{\partial t} = -DE + CE + CK \quad K_E = \frac{1}{2} \int [(\bar{w}')^2 + (\bar{w}'')^2] + [(\bar{u}')^2 + (\bar{u}'')^2] dM \quad (4.61)$$

where

$$\gamma = - \left(\frac{\theta}{T} \right) \left(\frac{P_r}{P} \right)^{\kappa} \left(\frac{\kappa}{P_r} \frac{\partial P_r}{\partial \theta} \right)$$

$$GZ = \int [\epsilon] [\bar{Q}] dM \quad (4.62)$$

$$GE = \int \gamma [\bar{Q}'] [\bar{T}'] dM \quad (4.63)$$

$$CZ = - \int (\bar{w})_{\theta} (\bar{\alpha})_{\theta} dM \quad (4.64)$$

$$CE = - \int [\bar{w}'] [\bar{\alpha}'] + \bar{w}'' \bar{\alpha}'' dM \quad (4.65)$$

$$CK = - \int [\bar{u}'] [\bar{v}'] + \bar{u}'' \bar{v}'' \frac{\cos \phi}{r} \frac{\partial}{\partial \phi} \left(\frac{[\bar{w}]}{\cos \phi} \right) dM \quad (4.66)$$

$$- \int [\bar{u}'] [\bar{w}'] + \bar{u}'' \bar{w}'' \frac{\partial}{\partial \phi} ([\bar{w}]) dM$$

$$CA = - C_p \int \left(\frac{\theta}{T} \right) [\bar{v}'] [\bar{T}'] + \bar{v}'' \bar{T}'' \frac{1}{r} \frac{\partial}{\partial \phi} \left(\frac{[\epsilon] [\bar{T}']}{[\theta]} \right) dM \quad (4.67)$$

$$- C_p \int \left(\frac{\theta}{T} \right) [\bar{w}'] [\bar{T}'] + \bar{w}'' \bar{T}'' \frac{\partial}{\partial P} \left(\frac{[\epsilon] [\bar{T}']}{[\theta]} \right) dM$$

$$DZ = - \int ([\bar{v}]) [\bar{F}_z] + [\bar{v}] [\bar{F}_y] dM \quad (4.68)$$

$$DE = - \int [\bar{u}'] [\bar{F}_x] + \bar{u}'' \bar{F}_x + \bar{u}'' \bar{F}_y + \bar{v}'' \bar{F}_y + \bar{v}'' \bar{F}_y dM \quad (4.69)$$

where r is the earth's radius, $dM = r^2 (\cos \phi / g) d\phi d\lambda dP$, λ is longitude, ϕ latitude.

$$\frac{\partial}{\partial t} \int \underbrace{\left(\frac{[u]^2}{2} \right)}_{(A)} dm + \int \underbrace{\left(\frac{[u]}{R} \right)}_{(B)} \underbrace{\left(\frac{1}{R} \frac{\partial R^2 [uv]}{\partial y} + \frac{\partial R [u\omega]}{\partial P} \right)}_{(C)} dm$$

$$- \int f [u] [v] dm + \int [u] [F_z] dm = 0 \quad (4.12)$$

$$\begin{aligned} \frac{\partial A_{ij}}{\partial t} = & \frac{1}{g} \int_S \int_{P_1}^{P_2} \underbrace{\{\epsilon q\} dP dS + \frac{1}{g} \int_S \int_{P_2}^{P_1} \underbrace{\{\epsilon \omega \alpha\} dP dS}_{(B)}}_{(A)} \\ & - \frac{C_p}{g} \int_S \int_{P_1}^{P_2} \underbrace{\left\{ \nabla_P \cdot (T' \nabla_P) + \frac{\partial}{\partial P} (\omega T') \right\} dP dS}_{(C)} \end{aligned} \quad (4.31)$$

$$[v] = \frac{\partial \psi}{\partial p} \quad \text{and} \quad [\omega] = - \frac{\partial \psi}{\partial y} \quad (6.46)$$

$$\frac{\partial [u_g]}{\partial p} = \frac{R p^{\kappa-1}}{f P_0^{\kappa}} \frac{\partial [\theta]}{\partial y} \equiv \gamma \frac{\partial [\theta]}{\partial y} \quad (6.49)$$

$$A \frac{\partial^2 \psi}{\partial y^2} + 2B \frac{\partial^2 \psi}{\partial y \partial p} + C \frac{\partial^2 \psi}{\partial p^2} + D \frac{\partial \psi}{\partial y} + E \frac{\partial \psi}{\partial p} = \gamma \frac{\partial H}{\partial y} + \frac{\partial \chi}{\partial p} \quad (6.50)$$

$$A = -\gamma \frac{\partial [\theta]}{\partial p} \quad B = \frac{\partial [u]}{\partial p} = \gamma \frac{\partial [\theta]}{\partial y} \quad C = f - \frac{\partial [u]}{\partial y}$$

$$E = - \frac{\partial \gamma}{\partial y} \frac{\partial [\theta]}{\partial y} \quad D = \frac{\partial \gamma}{\partial p} \frac{\partial [\theta]}{\partial y}$$

$$\chi = [F_z] + \frac{\partial [u'v']}{\partial y} + \frac{\partial [u'\omega']}{\partial p} \quad H = - \left(\frac{\partial [\theta'v']}{\partial y} + \frac{\partial [\theta'\omega']}{\partial p} \right) + \frac{[Q][\theta]}{[T]}$$

$$\frac{\partial \zeta_a}{\partial t} + \mathbf{V}_\psi \cdot \nabla \zeta_a = - \mathbf{V}_\chi \cdot \nabla \zeta_a - \zeta_a D \equiv S$$