

1. Mountain and Frictional Torques calculation over a sector.

Recall this equation (1.2) from Grotjahn (2016)

$$\int_0^{2\pi} \int_{-\pi/3}^0 \tau r^3 \cos^2(\phi) d\phi d\lambda = \int_0^{z_{top}} \int_{-\pi/3}^0 \{p_E - p_W\} r^2 \cos(\phi) d\phi dz$$

The equation was applied for motion in the Southern Hemisphere to estimate a pressure difference for cancellation between P change and surface stress. This problem uses the equation but evaluates both sides to see their contributions to torque applied to the atmosphere by the solid earth but for a sector including the Rockies. The sector ranges from 30N to 60N and 125W to 100W. The two sides **will not** be equal in this problem.

Use a mountain ridge line height of 2.5km. Assume the pressure decreases linearly as elevation increases above $z=0$, hence $F(z) = (2.5 \times 10^3 - z)/2.5 \times 10^3$ describes that vertical variation. Assume the zonal average meridional wind, $[v] = 0$.

Observed surface winds, east, and west pressures are:

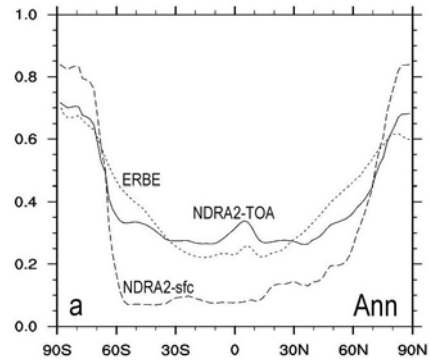
Latitude	30	35	40	45	50	55	60	degrees
[u]	0	2	5	8	10	8	5	m/s
p_E	1023	1025	1024	1022	1020	1018	1016	hPa @ $z=0$
p_W	1018	1018	1014	1012	1010	1008	1010	hPa @ $z=0$

Torque applied to the Earth is sought, so: $\tau = \rho C_D |\mathbf{V}| [u]$ (positive sign). Assume density is constant and equals the value at 800m when estimated using a scale height of 8km. Let $C_D = 2 \times 10^{-3}$ while density at $z=0$ is $\rho_s = 1.25 \text{ kg/m}^3$. r is given in Appendix B.

- (1 pt) calculate the density ρ to be used here.
- (1 pt) calculate $\rho C_D r^3 \Delta\phi \Delta\lambda$ where the longitude and latitude changes are in radians.
- (2 pts) Using part c, calculate the frictional torque (LHS) for the region using trapezoidal rule to solve the latitude integral. See: <http://grotjahn.ucdavis.edu/course/atm150/integral.htm> for more information. Evaluate the zonal integral analytically.
- (3 pts) Calculate the mountain torque (RHS) from the Canadian Rockies from these conditions. Assume the pressure difference decreases linearly with height according to $F(z)$. Use trapezoidal rule for the latitudinal integration. The integral over height is solved analytically. Caution: use Pa units for pressure.
- (2 pts) Compare your answers in parts c and d. Discuss which is larger and whether they are opposing or reinforcing each other. Frame your discussion (using complete sentences) in terms of the direction of each torque applied to the solid earth. Note: earth rotates eastward.
- (1 pt) It is sometimes assumed that the RHS is 10% (Volland, 1996) or 20% (Rosen, 1993) of the LHS. Why do you think this problem has a different result than this expectation? Hint, those statements arise in zonal and in global average context.

2. **Radiation** at various atmospheric locations, see table.

Location	TOA solar (W/m ²)	Absorbed solar (W/m ²)	Terrestrial (W/m ²)
North Pole	175	75	195
38 N	340	235	235
Equator	415	320	250



a. (6 pts) Calculate the average annual albedo at each of the 3 stations from this data. Obtain albedo values at these 3 locations from fig. 3.7a. (Use ERBE data.) Discuss similarities and/or differences between the 2 sets of estimates.

b. (10 pts) Assume that emissivity is constant and equals 0.9 for the spectrum of terrestrial emission. Calculate the implied annual mean temperatures for each location based on local radiative balance, i.e. upon using the absorbed solar radiation. Calculate the implied annual mean temperature based upon the terrestrial emissions. (There are 5 distinct values, since absorbed solar radiation and terrestrial emission are the same at 38N.)

c. (4 pts) Calculate the difference ($=\Delta T$) in temperature between using the actual emission minus temperature from solar absorption at the Equator and at the North Pole. What percent of the difference is due to heat transport at the two locations? 'Percent' means $\{\Delta T / \text{local solar absorption-based } T\} * 100\%$ so the percentage is <0 at the equator.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.

Rosen, 1993, DOI: 10.1007/BF01044076

Volland, 1996, DOI: 10.1007/BF01904476