ATM 240 – Fall 2016
Problem set #3 45 pts Due: 20 October 2016

1. Daily total Q_{Sol} insolation at latitude φ, for solar declination angle δ, for the half length of daylight (in radians) H_L , and assuming the Earth's distance from the Sun is a constant is given by:

$$Q_{Sol} = \frac{24(hrs)}{\pi} I_{Sol} \left(H_L \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin H_L \right)$$

(e.g. Peixoto and Oort, 1992). Where the length of daylight is given by

$$2 H_L = 2 \arccos\left\{ -\tan\varphi \tan\delta \right\}$$

(Note: if the sun does not rise or set at a location, the formula is undefined and $2H_L=2\pi$.) Consider three locations (equator and North Pole) at the summer solstice (δ=23.5 in degrees).
a. (1pt) Find δ in radians
b. (2 pts) Find the length of daylight at both locations. (show your work)
c. (4 pts) Find Q_{Sol} at both locations. Note that Q_{Sol} has units: W m^{-2} in 24 hours. All angles used should be in radians.
d. (10 pts) From the relation that $I_{IN} = a_{SG}\, Q_{Sol} / 24$, use the glass slab model equations to find T_A and T_G at both locations given: $a_{SG} = 0.75$ at the equator and =0.35 at the North Pole. Assume that $a_{SA} = 0.3$ and $a_{LA} = 0.9$ at both locations.
e. (3 pts) How do these T_G temperatures compare with figure 3.11b (Grotjahn, 1993)?

2. A simple linkage between the glass slab calculation, diabatic cooling, compressional heating, and thus the motions of a "Hadley-type" *polar* cell. At 80 N, let $a_{LA} = 0.85$, $a_{LG} = 1.0$, $a_{SA} = 0.3$. From figure 3.11a, b (Grotjahn, 1993) it may be assumed that $T_G = 250$ K in winter, 273 K in summer. Let $T_A = 225$ K in winter; 240 K in summer.
a. (8 pts) find I_G and I_A then find the I_{IN} needed for radiative balance in each season.
b. (3 pts) from figure 3.8a (Grotjahn, 1993), one may estimate the observed solar radiation absorbed at 80 N; call that I^*. In winter I^*=0, in summer let it = 171 W/m^2. The difference between I^* and I_{IN} is the amount of heat flux energy needed per unit area. Call that Δ; and find that value for each season.
c. (8 pts) Δ is a loss of energy per second per square meter. What rate of heating (K/s) in the whole *column* is would be needed to *balance* this loss of energy if no work is being done? Assume that the heating rate (dT/dt) is a constant in the vertical and the surface pressure is 1.013×10^5 Pa. Find this heating rate for both seasons. Hints: force equals mass times acceleration, and pressure is a force per unit area. Recall the first law of thermodynamics equation: $C_P\, dT/dt = \alpha\, dp/dt + D_J$ where D_J is a diabatic heating rate per unit mass (W/kg units). D_J is proportional to Δ. Check units for consistency.
d. (6 pts) The dry adiabatic lapse rate is given by g/C_p where g=9.8 m/s^2 and C_p = 1004 J/(K kg). If air in the column is sinking or rising, what value of vertical velocity (w, in m/s) must be present in winter and in summer so that the local change of temperature is zero? For a bit of realism, assume that the air has lapse rate Γ = 6 K/km. Ignore any horizontal heat transport. Hints: sinking adiabatically heats the air. Also, $\partial T/\partial t$ is now zero. Check your method to ensure units are consistent.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.