ATM 240 – Fall 2016

Problem set #5 40 pts Due: 3 November 2016

1. **Isentropic mass weighted view versus a simple zonal average**.
Let a perturbation pressure field be defined as: $P' = \sin(y)\{(1-z)\cos(x) - Qz\sin(x)\} - Syz$
Assume these quasi-geostrophic assumptions apply:
$\theta' = \partial P'/\partial z$ $v' = \partial P'/\partial x$ where primes denote nondimensional perturbations
dimensional (total) values are given by:
$\theta = \theta_s ( 1 + C * \theta')$ $P = P_s ( 1 + C * P')$ $v = 10 v'$ ( in m/s) $Z = z * 10$km
The subscript "s" denotes a static state (dimensional) which is isothermal with temperature $T_s$ =260K. The surface pressure is $P_s (0)$ = 1.e+5 Pa. C=0.02. Q=4. S=1. And scale height H (=$RT_s /g$) is constant.

a. (2 pts) Derive the formula for $\theta_s$ as a function of H, $T_s$, z and $\kappa$ (where $\kappa = R/C_p$ ). $P_s$ should not be in your final formula. Hint: $P_s = P_s (0) \exp(-Z/H)$ where if this Z is in km, then so must be H. Hint: you can check your answer: θs=265K is about 500m and θs=364 is about 9km.

b. (4 pts) Derive the formulae for v' and θ' as general functions of (x, y, z).

c. (10 pts) Make contour plots of: P', v', and θ' for z=0 and for y = (0, π) and x = (0, 2π). Do the same for Z=9km (z =0.9). Use 21 pts in x range; 11 pts in y range. What is the value of H? These plots have P', θ', v' consistent with a developing baroclinic wave and geostrophic meridional wind. Discuss: a) P' trough location changes with height, b) where in x are: the coldest air, the upper trough, and lower trough centered, and c) which level has larger v'θ'.

d. (2 pts) Calculate a zonal average of v' at (y, z) =(π/2,0.9). You may solve this numerically using the 20 grid points in the zonal direction. Trapezoidal rule is ok. You can assume that the domain is periodic. Hint: don't double count the end point.

e. (4 pts) Derive the formula for the height of a potential temperature surface $\theta_o$ . Test out your formula by plotting the height of the q=265K surface for x = (0, 2π) and y = (0, π). Hint: your plot should look similar to fig. 3.18 if that figure ranged from -1 to 5 km (instead of 0-10 km).

f. (8 pts) Plot v' for θ=265K and θ=364K surfaces. Set the v'=0 for any point that is underground (i.e. having elevation <0).

g. (10 pts) Calculate 2 mass-weighted zonal averages at y=π/2 of v': one on the q=265K surface and one on the θ=364K. A simple mass weighting starts with the z value at each grid point of the θ surface.. Next find the $P_s$ of each z value, next multiply the v' value by the $P_s$ value sum the corresponding values and call that the v'$P_s$ sum. Next calculate the corresponding sum for the Ps values of the grid points used. Finally divide the v'$P_s$ sum by the Ps sum. Note: **do not include any points in either sum that are underground**. These upper and lower elevation mass-weighted averages will be consistent in sign with the poleward extension of a Hadley cell seen in θ coordinates. Include a labeled table of these z, $P_s$ , and v'*$P_s$ values for each θ surface

**NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any table or plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.**