ATM 240 – Fall 2016

Problem set #6 30 pts Due: 10 November 2016

1. **Eddy momentum fluxes**. Assume that a streamfunction field Ψ is defined:

$$\Psi = \sin(x + A\,y)\cos(y)\;\; - \sin(By)$$

Where $\pi/2 \le x \le 2\pi + \pi/2$ and $-\pi/2 \le y \le \pi/2$. Also, A and B are constants. Note that $u = -\partial\Psi/\partial y$ and $v = \partial\Psi/\partial x$. To add additional interest, leave A and B as a variable in your solution and in your plotting program (be it FORTRAN, matlab, excel, etc.) Use circled values of A and B:

Let A = -0.9, -1.5, -0.5, -0.5, 0.5, 1.0, -0.3 and B = 1.2, 1.5, 1.5, 1., 1., 1.5, 1.5

a. (8 pts) Derive the general form of the zonal average of the eddy momentum flux: [uv]. Solve all integrals (it simplifies greatly). Hint: your result should only be a function of A and y.

b. (3 pts) Derive the formula for zonal mean eddy momentum convergence, -d[uv]/dy.

c. (4 pts) Make contour plots of Ψ and uv for the A and B specified over the x and y ranges given. Use 21 grid points in y and 41 grid points in x. Make sure axes and contour values are clearly labeled.

d. (5 pts) Make a single plot containing: [Ψ], [u], [uv], and d[uv]/dy for the A and B specified over the y range given. Use 21 grid points in y. Make sure axes and lines are clearly labeled.

2. **Kinetic energy maintenance.** Conceptually, zonal average momentum convergence should create zonal average westerly acceleration. However, plots of observed [u'v'] have maxima quite close to the observed maxima of [u] leading to eddy momentum convergence ( $\partial$[u'v']/$\partial\varphi$ >0) far poleward of the [u] maximum. Careful inspection of the KE equation finds cosine factors that bring the picture into consistency with intuition.

a. (2 pts) Imagine the distribution of vertical and time average [u] is given by: {[u]}= 20*sin(2.9φ-π/12) and that the distribution of the time and vertical average of [u'v'] is given by: {[u'v']}=22*sin(2.9φ-π/12). (Yes, nearly the same function.) Find the numeric values for {[u]} and {[u'v']} for the latitude range 10N to 70N. Use a 2.5 degree interval.

b. (2 pts) Find the numeric values of {[u]}/cos(φ) and {[u'v']}*$\cos^2$(φ) for 10N to 70N range with 2.5 degree interval.

c. (1 pt) Find (-0.04)*{[u]}/cos(φ)*$\partial$({[u'v']}*$\cos^2$(φ) )/$\partial\varphi$ for the 12.5N to 67.5N latitude range (2.5 degree interval) using finite differences,. Hint, φ is in radians in your finite difference.

d. (5 pts) On one chart, plot the 5 quantities calculated here in parts a, b, and c, for the 15N to 65N latitude range. Comment on the relative locations of the peak values of these 5 quantities. How well does momentum convergence match the inverse cosine-weighted zonal mean zonal flow? Make sure all aspects of your chart are clearly labeled.

**NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.**