ATM 240 – Fall 2016

Problem set #8 24 pts Due: 29 November 2016*

1. Eddy fluxes create meridional cells. The Kuo-Eliassen equation (K-E Eqn) models the mean meridional circulation. The K-E Eqn is simplified below in order to facilitate finding a solution using a double Fourier series. One simplification assumes that the earth has such a strong meridional circulation that the meridional gradient of [Θ] is negligible and can be assumed to be zero. The K-E Eqn is linear so one may find the circulation created by each forcing. The eddy momentum forcing by eddies is considered here. After choosing appropriate scales of motion, these assumptions are consistent with the following Cartesian form of the Kuo-Eliassen equation:

$$A\, \partial^2\Psi/\partial y^2 + C\, \partial^2\Psi/\partial p^2 = \partial\chi/\partial p \qquad (1)$$

where A=9. and C=1.2 and are constants. The meridional derivative of [u'v'] is specified by:

$$\chi(y,p) = \{\, dq(y)/dy \,\}*\{1 + \cos(\pi p)\,\} \qquad (2)$$

where q(y) will be specified as:

$q = \{s\, y\}\, \gamma$ for $0. \le y \le 2.$

$q = \{2s + g*(2 - y)\}\, \gamma$ for $2. \le y \le 4.$

$q = \{2s - 2g + h*(y-4)\}\, \gamma$ for $6. \ge y \ge 4.$

The domain in p is from p=1.0 to p=0. where p has no units and p=1 is the bottom of the atmosphere. The domain in y is from 0 to 6, where the "equator" is at y=0 and y=6 is the "north pole". The stream function, Ψ must be zero along the boundaries of the domain.

a. (4 pts) Solution form (3) can be assumed for (1). Verify that (3) satisfies the boundary conditions where n and m are integers. $F_{n,m}$ does not depend on y or p.

$$\Psi(y,p) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_{n,m} \sin\left(\frac{n\pi y}{6}\right)\sin\left(m\pi p\right) \qquad (3)$$

b. (4 pts) Use a computer program to plot a diagram showing q as a function of y. Let s=2., g=2.5, h=0.5. Let γ =1. In the same program, make a contour plot of [u'v'] = q*{1+cos(πp) for the entire p and y range. Use 61 points in y and 21 in p. The contour plot should have pattern roughly similar to observations (e.g. Fig.4.12e)

c. (3 pts) Explain why the only relevant value of m in (3) will be m=1 for this problem. Do so by solving the orthogonality condition of the Fourier series basis functions of the series for the p variation. Show how the orthogonality greatly simplifies the calculation of the Fourier coefficients for the information given.

d. (9 pts) Find the analytic, general formula for the Fourier sine coefficients $F_{n,1}$ for this "January" case. Evaluate all integrals and derivatives as encountered. The final answer should contain no integrals or derivatives. Write out and simplify the specific formulas for $F_{1,1}$, $F_{2,1}$, and $F_{3,1}$. All formulas should have s, g, h, A, C, and γ **un**specified. Hint: express the RHS of (1) as a Fourier series in sin(nπy/6) and use the orthogonality.

e. (4 pts) Use a computer program to make a contour plot of the ψ field. Be sure your plot is clearly labeled along axes as well as contour values. Let s=2., g=2.5, h=0.5, and γ=1. Let A=9. and C=1.2. Use 61 grid points in y and 21 grid points in p. Choose a small enough contour interval to make the 3 meridional cells visible. It is sufficient to terminate the summation of n at 3. (Include higher values of n, if you wish.)

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.