ATM 240 – Fall 2019

Problem set #8 20 pts Due: 3 December 2019

1. **(Simplified) Kuo-Eliassen equation** (K-E Eqn) model of the mean meridional circulation. The K-E Eqn is simplified below in order to facilitate finding a solution using a double Fourier series. One simplification assumes that the earth has such a strong meridional circulation that the meridional gradient of [Θ] is negligible and can be assumed to be zero. It is further assumed that latent heating dominates the circulation forcing (i.e. the forcing is largest in the middle troposphere of the tropics). Assume that forcing by eddies and friction is negligible. After choosing appropriate scales of motion, these assumptions are consistent with the following form of the Kuo-Eliassen equation:

$$A\ \partial^2\Psi/\partial y^2 + C\ \partial^2\Psi/\partial p^2 = \partial H/\partial y \qquad (1)$$

where $A=16a/\pi^2$ and $C=0.81b/\pi^2$ and a and b are constants. The diabatic heating H is specified by:

$$H(y,p) = h(y)\sin(\pi(P-0.1)/0.9) \qquad (2)$$

where h(y) will be specified as:

$h = \{d + e\,y\}\,\gamma$ for $-2. \le y \le -0.5$

$h = \{d + e\,y\}\,\gamma$ for $-0.5 \le y \le 0.5$

$h = \{f + s\,y\}\,\gamma$ for $2. \ge y \ge 0.5$

The domain in P is from P=1.0 to P=0.1 where P has no units and P=1 is the bottom of the atmosphere. The domain in y is from -2 to 2, where the equator is at y=0. The stream function, ψ is zero along the boundaries of the domain.

a. (4 pts) Solution form (3) can be assumed for (1). Verify that (3) satisfies the boundary conditions where n and m are integers.

$$\Psi(y,P) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_{n,m}\ \sin\left(\frac{n\pi(y+2)}{4}\right)\sin\left(\frac{m\pi(P-0.1)}{0.9}\right) \qquad (3)$$

b. (2 pts) Explain why the only relevant value of m in (3) will be m=1 for this problem. Hint: the othogonality of the basis functions in a Fourier series greatly simplifies the calculation of the Fourier coefficients for the information given.

c. (6 pts) Find the analytic formula for the Fourier sine coefficients $F_{n,m}$ for this case. The formula should have a, b, d, e, f, s, and γ unspecified. Evaluate integrals and derivatives as needed. The final answer should contain no integrals.

d. (2 pts) Use a computer program to plot a diagram showing h as a function of y. Let d=0.3, e=0.5, f=1.5, s= -1.5 and γ =1.

e. (4 pts) Use a computer program to make a contour plot of the ψ field. Be sure your plot is clearly labeled along axes as well as contour values. Let a=1., b=1. and γ=1. Let d=0.3, e=0.5, f=1.5, s= -1.5. Write a computer program (or use software like R, Excel, Mathematica) to make the plot. A pleasing plot can be made by using 41 grid points in y and 21 grid points in P. Choose a small enough contour interval to make any "Hadley" cells visible. It is sufficient to terminate any summation of n or m after 2. (Depending on the software used, you may find it easy to include a higher number of meridional wavenumbers, n, and if so, you are strongly encouraged to do so.)

f. (2 pts) Describe the direction your stream function field circulates (clockwise or counter clockwise) in each 'cell'. Describe how your stream function field relates to properties of h.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

Any plot must be completely and unambiguously labeled, including title and axes.