

Reality is not an exhibit for man's inspection, labelled "Do not touch."

Science, like art, is not a copy of nature but a recreation of her.

We remake by the act of discovery, in the poem or in the theorem.

Bronowski, 1956

Appendix A

Averaging

Purpose: Averages and deviations there from are defined and combinations used to isolate, approximately, various phenomena.

Bronowski could have been talking about the act of investigation that goes *before* the discovery. To some extent the type of answer one gets is limited by the way in which the question is asked. To discuss the "General Circulation" implies the use of a model: that global scale motions are emphasized while global patterns of radiation, moisture, and chemistry are included only as they relate to the dynamics ("circulation"). Partitioning the atmosphere this way defines and limits the questions and answers allowed on the topic.

In the subject of the general circulation this is most apparent when the question is how to divide up the atmosphere. There are many ways to divide a pie, some are better than others.

Evidence can be found in this book that dividing along zonal average versus nonzonal average has some limited utility when applied to the real atmosphere. The division is made for several reasons, but two are most important. First, observations and theories available in the literature on the General Circulation are often organized around zonal averages and deviations. So, one may not have a choice. Second, zonal averaging removes one dimension and time averaging tends to select the net effect of a lot of variation. The result is a two-dimensional picture that is easier to draw and grasp on a flat page than would be a four-dimensional image.

Sometimes the averaging makes the mathematics easier and the relationships between variables more clear. So, the essential elements of the atmosphere are more easily seen.

The act of dividing atmospheric properties into groups of zonal averages and deviations is to employ a model. In doing so, one emphasizes a distinction between the average and deviation that may not be the best model of reality.

A particular model is a specialized tool in the way that a screwdriver is. A screwdriver can turn screws but has limited use as a hammer. More to the point, with a screwdriver one tends to connect objects by attaching screws rather than hammering nails. The connections drawn throughout this book can viewed in a similar light.

A.1 Basic definitions

Various types of averages are applied throughout the book. The purpose is often to isolate various types of phenomena. Researchers typically average in horizontal space directions, in time and occasionally in the vertical. This appendix defines the convention of symbols used everywhere in this text.

Square brackets denote an average over longitude.

$$[q] = \frac{1}{2\pi r \cos \varphi} \int q dx$$

where q is any dependent variable and $2\pi r \cos \varphi$ is the circumference of a latitude circle for a complete zonal average. A deviation from this average is denoted by a single prime superscript:

$$q' \equiv q - [q].$$

An overbar indicates time averaging.

$$\bar{q} = \frac{1}{\tau} \int_{t_1}^{t_2} q dt$$

where $\tau = \int_{t_1}^{t_2} dt$. Departures from a time average are denoted by a double prime superscript:

$$q'' \equiv q - \bar{q}.$$

A curly overbar specifies vertical averaging.

$$\overline{q} = \frac{1}{P_0} \int_0^{P_0} q dP$$

where P_0 is the surface pressure, and pressure is the vertical coordinate. Departures from a vertical average are indicated with an asterisk: $q^* \equiv q - \overline{q}$.

A.2 Some general rules

An average of a single quantity's deviation from that average is zero. In other words, $[q']$ equals zero by definition, but $[q'']$ is not necessarily zero. The average of the product of two deviations is not necessarily zero either: $[q'q''] \neq 0$ unless $q' = 0$ everywhere. The average of a product of an average times its deviation can be simplified since the average can be brought outside the relevant integral. For example: $[[q]q'] = [q][q'] = 0$. Different averages are not always independent though that assumption is occasionally made in this text for convenience, i.e.,

$\overline{[q]} \neq \overline{[q]}$; here the vertical average includes P_0 , which may be a function of longitude.

Finally, successive averages are redundant.

Averages can be combined in several ways to partition a field. To illustrate the phenomena selected by each combination of averages and deviations, a longitudinal average is made first, then time, then pressure.

$$\begin{aligned} v &= [v] + v' \\ v &= \overline{[v]} + [v]'' + \overline{v'} + (v')'' \\ v &= \underbrace{\overline{[v]}}_{(1)} + \underbrace{\overline{[v]}^*}_{(2)} + \underbrace{[v]''}_{(3)} + \underbrace{([v]''^*)}_{(4)} + \underbrace{(\overline{v'})}_{(5)} + \underbrace{(\overline{v'})^*}_{(6)} + \underbrace{(v')''}_{(7)} + \underbrace{((v')'')^*}_{(8)} \end{aligned}$$

Each numbered term tends to select for a specific process:

1. A net average velocity; essentially a net mean mass shift or flux.

2. The time-mean meridional cell velocity, such as the time mean Hadley cell. The only variations allowed are in the vertical and meridional.
3. Time-varying, zonal average mass flux.
4. Time-varying meridional cell flux.
5. Flux due to standing barotropic eddies (only zonal variation is allowed, but no time variation).
6. Flux due to standing baroclinic eddies (must have vertical and zonal variations, but no time variation).
7. Flux due to transient barotropic eddies.
8. Flux due to transient baroclinic eddies.

Mean meridional cells are the circulations (like the supposed Hadley and Ferrel cells) that do not vary with longitude but do vary with height. The eddies include all phenomena that vary with longitude. If these eddies vary with height, they are “baroclinic”; if they do not, they are “barotropic.” To be strictly proper, baroclinic and barotropic labels should refer to either the type of energy conversion that predominates for a given eddy or some other property, such as the alignment of the mass and thermal fields. However, the structure adopted by an eddy is different for the two modes of energy conversion. In a crude manner, a vertical average will often select mainly storms that have the structure associated with the barotropic conversion. Similarly, calculating vertical deviations tends to select the baroclinically developing eddies. “Transient” phenomena are those that vary with time, while “standing” phenomena are time-invariant.

Phenomena can be gathered into groups by selectively combining and applying the averages and deviations. For example, all eddy phenomena can be isolated by merely selecting deviations from a zonal average. To see more sophisticated examples of this grouping, all three averages are applied to the velocity covariance uv , where u is zonal and v is meridional velocity. It is useful to break up this covariance into different contributions. The resultant phenomena being isolated depend a lot upon the order in which the averages are applied.

Various partitions of $\overline{[uv]}$ are considered. A zonal, then a time, and finally a pressure average are taken. The steps may be easier to follow if one takes an indirect approach. Partitioning u into zonal mean and zonal deviation first:

$$u = [u] + u'$$

Next, a time partitioning, but only to the *first* term on the right hand side.

$$[u] = \overline{[u]} + [u]''$$

The first term on the right side of this equation is split according to the pressure average:

$$\overline{[u]} = \overline{[u]} + \overline{[u]}^*$$

Now substituting for the first term in each of these three equations obtains:

$$u = \overline{[u]} + \overline{[u]}^* + [u]'' + u' \quad (A.1)$$

The same set of operations for v yield:

$$v = \overline{[v]} + \overline{[v]}^* + [v]'' + v' \quad (A.2)$$

Now that u and v are expanded, (A.1) and (A.2) are multiplied and then the averages are again applied to all the terms. Many of the terms cancel. All terms involving the product of a mean and its deviation vanish. For example, when three averages are applied:

$$\overline{\overline{[u][v]''}} = \overline{[u]} \overline{[v]''} = 0$$

Similarly, other mixed terms vanish. One is left with products of “like” terms. That is:

$$\overline{[uv]} = \underbrace{\overline{[u][v]}}_{(1)} + \underbrace{\overline{[u][v]}}_{(2)} + \underbrace{\overline{[u]^* [v]^*}}_{(3)} + \underbrace{\overline{[u]'' [v]''}}_{(4)} + \underbrace{\overline{[u'v']}}_{(5)} \quad (A.3)$$

Multiplying by the proper factor ($2\pi r \cos \phi \tau P_0/g$), the five terms in (A.3) become mass fluxes.

One can interpret (A.3) in light of the physical phenomena isolated by each term. In order to understand better how the interpretation is deduced, *one always works from the innermost operation (be it an average or deviation) toward the outermost operation prior to the multiplication*. Using term (4) as an example, the innermost operation is a zonal average; so the phenomena must not vary with longitude. The next operation is a time deviation, hence the phenomena must be zonally symmetric and vary with time. The quantities are next multiplied before any further averages are taken so there is no further restriction upon the phenomena included in term (4).

- *Term (1)* is the flux of relative angular momentum through a vertical “wall” around a latitude circle and during the time τ .
- *Term (2)* is that part of the angular momentum flux due to a *net mass shift*.
- *Term (3)* isolates phenomena that have departures from the vertical of the time and zonal averages. This term isolates the *time mean or climatological mean meridional circulation*.
- *Term (4)* is either or both of (a) the *time varying meridional cells* and (b) the time variation of the *net mass flux*.
- *Term (5)* selects all transverse waves. It includes both (a) *standing and transient barotropic eddies* and (b) *standing and transient baroclinic eddies*.

These brackets, bars, and primes are sometimes confusing when seen for the first time. Much of the mystery about how to use these symbols can be removed if one remembers that these symbols merely represent integrals and local subtractions. The symbols for averages are integrals. The symbols for deviations denote subtracting the integrated value from the actual value at a local point.