Appendix D

Fundamental Radiation Concepts

Purposes: Provide foundation concepts and equations as a starting place for discussions of radiation in the book.

Atmospheric radiation is expressed using radiance (E) and irradiance (I). I is the total amount of radiant energy actually passing through (or arriving at) an area. E is the amount of I passing through (or arriving at) an area in (from) a particular direction. Hence, I and E are related by geometry. Radiance units are usually energy/(unit time x unit area x unit solid angle for a wavelength) so E has units $W/(m^2\ sr)$ where sr is steradians. The relation to irradiance depends on whether radiance varies with direction, and how many directions the radiation is occurring. For emission from a plane (parallel) surface, the emission would be in all directions of a hemisphere. Both radiance and irradiance units depend on whether the energy flux is expressed for a single electromagnetic spectrum wavelength (Λ) or integrated over all such wavelengths; in the latter case, the units are usually W/m^2.

To relate radiance and irradiance, note that I arriving at a unit area on a horizontal plane is the total radiant energy *normal* to the unit area of the plane. See figure D.1. To calculate I requires a $\cos\varphi$ factor (explained in a moment) where φ is the zenith angle. Thus:

$$I = \int_o^{2\pi} E(\varphi,\lambda)\cos\varphi\, d\omega \qquad \text{(D.1)}$$

The unit solid angle is related to changes in azimuth ($d\lambda$) and zenith angle ($d\varphi$) by

$$d\omega = \sin\varphi\, d\lambda\, d\varphi \qquad \text{(D.2)}$$

where the $\sin\varphi$ factor arises from "convergence of meridians" on a sphere. Combining (D.1) with (D.2) gives

$$I = \int_0^{2\pi}\int_0^{\pi/2} E(\varphi,\lambda)\cos\varphi\,\sin\varphi\, d\varphi\, d\lambda \qquad \text{(D.3)}$$

Note that (D.3) is for radiation over one hemisphere only. Examples include radiation reaching the ground from: infra-red emission by the atmosphere, or *scattering* of solar radiation by the atmosphere. In some problems, one wants the radiation from the whole sphere: an example would be the radiation absorbed by a volume of air, where the radiation comes from above and below; however, it is common to still use hemispheres and separate the radiation coming from below ('up') from that coming from above ('down').

If the radiation is isotropic (constant in all directions) then E is independent of λ and φ; E can be brought outside the integral in (D.3). The result is (D.3) reduces to $I = \pi E$. In Figure D.1 this is the irradiance in the downward direction. Since E is isotropic, the radiant energy passing through any solid angle $d\omega_1$ equals the energy passing through any other solid angle $d\omega_2$ of the same size.

The $\cos\varphi$ factor in (D.1) is explained as follows. Figure D.2 shows isotropic *radiance* from an infinite plane reaching a hemisphere below. The flux through each unit solid angle $d\omega_1$ and $d\omega_2$ is the same because the radiance is isotropic. Those solid angles are oriented perpendicular to the radiation from any direction φ but a horizontal surface is not perpendicular to the radiation from any direction except the direction perpendicular to that surface ($\varphi=0$). In Figure D.2 the emitting areas A_1 and A_2 that emit radiant energy towards $d\omega_1$ and $d\omega_2$ are *not* equal areas. Instead, $A_2 = A_1 / \cos(\varphi)$ so a larger emitting area (A_2) in figure D.2 is needed for the radiance striking the hemisphere to be isotropic. In terms of energy reaching a *horizontal* surface, energy approaching from a nearly horizontal angle ($\varphi \sim \pi/2$) has almost no amount perpendicular to the horizontal surface because radiation from that direction is nearly parallel to the horizontal surface. Hence, the radiance striking a horizontal surface from a particular direction φ will be proportional to the radiance E from that direction times $\cos(\varphi)$. Since the $\cos\varphi$ factor is a result of the geometry, it also applies for non-isotropic radiation.

The relationship between I and E simplifies for parallel beam radiation. Solar radiation comes from a small solid angle $d\omega_m$ only, then the integral over $d\omega$ is very small, too. The integral over solid angle can be approximated numerically as a finite sum over many individually small solid angles, only one of which has the size and is in the direction of the Sun. For solar radiation, then the integral (D.1) reduces to:

$$I = \sum_{i=1}^{N_{d\omega}} E_i \cos\varphi_i \, d\omega_i = E_m \, d\omega_m \tag{D.4}$$

Consequently, one can dispense with the solid angle integration and just use I to characterize direct solar radiation. However, terrestrial radiation (infra-red) and scattered solar radiation come from many angles and integration over solid angles and using radiance E are required.

Planck's law of blackbody radiation defines the *monochromatic radiance,* E_Λ (or *spectral radiance*) as the amount of radiant energy flux passing through a given area at a wavelength (Λ) of electromagnetic radiation for a given blackbody temperature (*T*) from a specific direction. The units of E_Λ are commonly W/(sr m^2 nm) where the length scale nm has scale similar to the wavelength of the radiation Λ. Planck's law may be written

$$E_\Lambda = \frac{2 c_h c^2}{\Lambda^5 \left\{ \exp\left(\frac{c_h c}{c_B \, \Lambda \, T} \right) - 1 \right\}} \tag{D.5}$$

where T is the temperature of the black body emitter, lambda (Λ) is the wavelength of light, c_h = 6.625 x 10^{-34} Js is the Planck constant, c = 3 x 10^8 m/s is the speed of light, c_B = 1.38 x 10^{-23} J/K is the Boltzmann constant.

Planck's law contains two functions of Λ. As Λ becomes smaller, Λ^5 decreases but the term in the curly brackets becomes larger more rapidly making (D.5) decrease to zero as Λ goes to zero. At very short wavelengths: for decreasing wavelength, the variation with wavelength in square brackets (which is decreasing) dominates the minus fifth power out front (which is increasing). In that case the exponential is much larger than one and an approximate form of (D.5) is:

$$E_\Lambda \approx 2c_h c^2\, \Lambda^{-5} \exp\left(\frac{-c_h\, c}{c_B\, \Lambda T}\right) \tag{D.6}$$

Relation (D.6) approximates Planck's law well if $c_h c >> c_B\, \Lambda\, T$. For very large values of Λ, both functions of Λ in the denominator of (D.6) become large and so E_Λ again approaches zero. Therefore, (D.5) indicates that the irradiance must reach a maximum at some wavelength.

The location of the maximum is a function of temperature. Taking a derivative of (D.6) w.r.t. Λ and setting the result to zero obtains a wavelength Λ_m where the emission is a maximum for this approximation. The resultant linear relationship between temperature T and wavelength of maximum emission Λ_m is known as Wien's law:

$$T\, \Lambda_m = 2.898 \times 10^{-2} \quad K\, m \tag{D.7}$$

Wien's law shows how the peak emission moves to a longer wavelength for a cooler temperature of the emitter.

Planck's law also shows that the higher the temperature, the closer the argument of the exp function in (D.5) approaches zero, hence the curly brackets in (D.5) approaches zero, and the larger the values of E_Λ at all wavenumbers..

One can formulate Planck's law using frequency by first noting that $c = \Lambda \upsilon_\Lambda$ where υ_Λ is the electromagnetic frequency at that wavelength. Since υ_Λ decreases when Λ increases an increment of radiance in wavelength is minus an increment in frequency, hence:

$$E_\Lambda(\Lambda, T)\, d\Lambda = -E_{\upsilon_\Lambda}(\upsilon_\Lambda, T)\, d\upsilon_\Lambda \tag{D.8}$$

Rearranging (D.8) and evaluating $-d\Lambda / d\upsilon_\Lambda = c / \upsilon_\Lambda{}^2$ obtains the alternate Planck's law

$$E_{\upsilon_\Lambda} = \frac{2c_h\, \upsilon_\Lambda{}^3}{c^2 \left\{ \exp\left(\frac{c_h\, \upsilon_\Lambda}{c_B\, T}\right) - 1 \right\}} \tag{D.9}$$

Integrating (D.9) over all frequencies and over all solid angles finds the total irradiant power per unit area, after use of the identity

$$\int_0^\infty \frac{x^3}{\exp(x) - 1} dx = \frac{\pi^4}{15} \tag{D.10}$$

obtains the Stefan-Boltzmann law:

$$I = \sigma T^4 \tag{D.11}$$

where I here is the total blackbody irradiance and $\sigma = (2\pi^5 c_B{}^4)/(15c^2 c_h{}^3) = 5.67 \times 10^{-8}\, W\, m^{-2}\, K^{-4}$.

‘Blackbody’ is a descriptive term to visualize the appearance of a ‘perfect’ emitter. According to (D.11), a ‘blackbody’ object emits I amount of radiant energy per unit area when it has temperature T. This ‘object’ can be many different things: atmospheric gas molecules, snow covered ground, ocean surface, etc. in our applications. However, not all objects are ‘perfect’ emitters.

The ability to emit radiation is measured by a parameter called the emissivity, ε_Λ . Where ε_Λ ranges from 0 (unable to emit radiation at wavelength Λ) to 1 (maximum emission). Absorptivity (a_Λ) is similar to emissivity except that it refers to the ability of that object to absorb radiation at wavelength Λ. Kirchhoff’s law states that $a_\Lambda = \varepsilon_\Lambda$. Hence, a good emitter is a good absorber at the same wavelength, similarly for poor emitter. ε_Λ can have quite different values for different wavelengths. For example, the absorptivity of clouds in visible light is low (for a thick cloud, ε_Λ < ~0.2), but for infrared wavelengths it is much larger (for a thick cloud, ε_Λ > ~0.8).

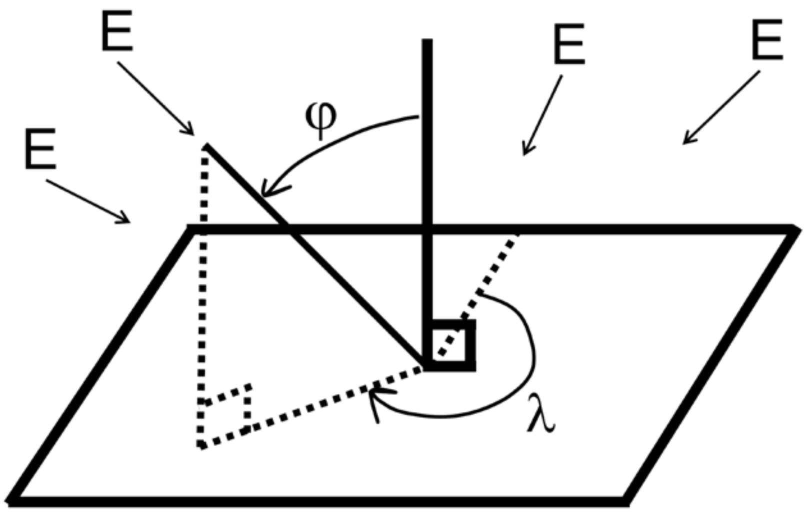

Figure D.1

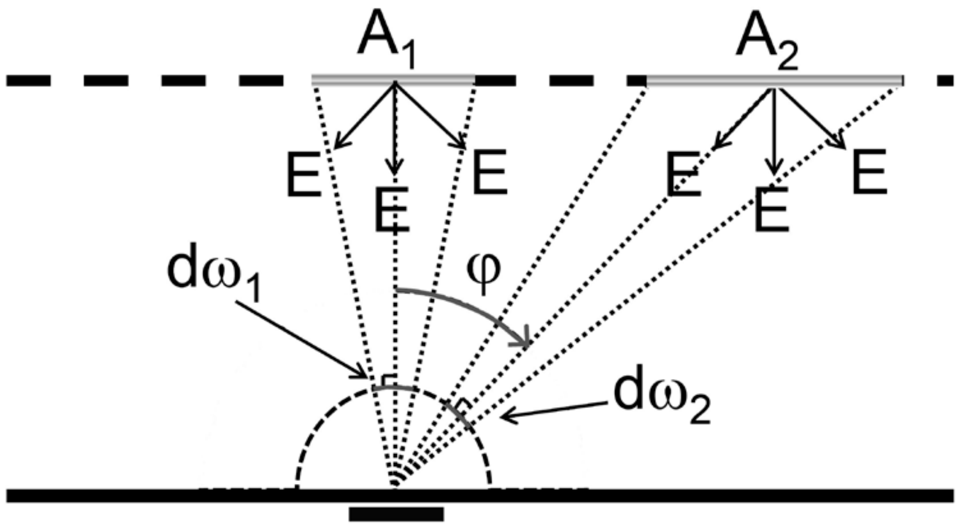

Figure D.2