if we call light, those rays which illuminate objects, and radiant heat, those which heat bodies, it may be inquired whether light be essentially different from radiant heat?

William Hershel, 1817

Seas, lakes and great bodies of water, agitated by winds, continually change their surfaces; the cold surface in winter is turned under, by the rolling of the waves, and a warmer turned up; in summer, the warm is turned under, and colder turned up. Hence the more equal temper of sea-water, and of the air over it.

Franklin, 1765

Chapter 3
Radiation and Temperature

Purpose: Describe radiant energy input, reflected, absorbed, and emitted and corresponding transports of heat and energy balance. Emission depends on an associated distribution of temperature whose structure is shown. Simple models explain the general properties.

The exploration of the observed general circulation begins with the ultimate source of that circulation: the complicated distributions of absorbed and emitted radiation. In chapter 1 the zonal and annual average distribution of absorption and emission were described in general terms (recall Figure 1.1); in this chapter radiation and its links to other variables are examined in more detail.

3.1 Radiation

3.1.1 Fundamental Equations

Atmospheric radiation is most commonly expressed using radiance (E) and irradiance (I). I is the total amount of radiant energy actually passing through (or arriving at) an area. On the other hand, E is the amount of I passing through (or arriving at) an area in (from) a particular direction. Hence, I and E are related by geometry. Irradiance units are usually energy/(unit time x unit area x unit solid angle for a wavelength) so E has units W/(m² sr) where sr is steradians. The relation to irradiance depends on whether radiance varies with direction, and how many directions the radiation is occurring. For emission from a plane (parallel) surface, the emission
would be in all directions of a hemisphere. Both radiance and irradiance units depend on whether the energy flux is expressed for a single electromagnetic spectrum wavelength ($\lambda$) or integrated over all such wavelengths; in the latter case, the units are usually W/m².

To relate radiance and irradiance, note that $I$ arriving at a unit area on a horizontal plane is the total radiant energy normal to the unit area of the plane. See figure 3.1. To calculate $I$ requires a $\cos \phi$ factor (explained in a moment) where $\phi$ is the zenith angle. Thus:

$$I = \int_0^{2\pi} E(\phi, \lambda) \cos \phi \, d\omega$$

(3.1)

The unit solid angle is related to changes in azimuth ($d\lambda$) and zenith angle ($d\phi$) by

$$d\omega = \sin \phi \, d\lambda \, d\phi$$

(3.2)

where the $\sin \phi$ factor arises from "convergence of meridians" on a sphere. Combining (3.1) with (3.2) gives

$$I = \int_0^{2\pi} \int_0^{\pi/2} E(\phi, \lambda) \cos \phi \sin \phi \, d\phi \, d\lambda$$

(3.3)

Note that (3.3) is for radiation over one hemisphere only. Examples include radiation reaching the ground from: infra-red emission by the atmosphere, or scattering of solar radiation by the atmosphere. In some problems, one wants the radiation from the whole sphere: an example would be the radiation absorbed by a volume of air, where the radiation comes from above and below; however, it is common to still use hemispheres and separate the radiation coming from below (‘up’) from that coming from above (‘down’).

If the radiation is isotropic (constant in all directions) then $E$ is independent of $\lambda$ and $\phi$; $E$ can be brought outside the integral in (3.3). The result is (3.3) reduces to $I = \pi E$. In Figure 3.1 this is the irradiance in the downward direction. Since $E$ is isotropic, the radiant energy passing through any solid angle $d\omega_1$ equals the energy passing through any other solid angle $d\omega_2$ of the same size.

The $\cos \phi$ factor in (3.1) is explained as follows. Figure 3.2 shows isotropic radiance from an infinite plane reaching a hemisphere below. The flux through each unit solid angle $d\omega_1$ and $d\omega_2$ is the same because the radiance is isotropic. Those solid angles are oriented perpendicular to the radiation from any direction $\phi$ but a horizontal surface is not perpendicular to the radiation from any direction except the direction perpendicular to that surface ($\phi=0$). In
Figure 3.2 the emitting areas $A_1$ and $A_2$ that emit radiant energy towards $d\omega_1$ and $d\omega_2$ are not equal areas. Instead, $A_2 = A_1 / \cos(\varphi)$ so a larger emitting area ($A_2$) in figure 3.2 is needed for the radiance striking the hemisphere to be isotropic. In terms of energy reaching a horizontal surface, energy approaching from a nearly horizontal angle ($\varphi \sim \pi/2$) has almost no amount perpendicular to the horizontal surface because radiation from that direction is nearly parallel to the horizontal surface. Hence, the radiance striking a horizontal surface from a particular direction $\varphi$ will be proportional to the radiance $E$ from that direction times $\cos(\varphi)$. Since the $\cos\varphi$ factor is a result of the geometry, it also applies for non-isotropic radiation.

The relationship between $I$ and $E$ simplifies for parallel beam radiation. Solar radiation comes from a small solid angle $d\omega_m$ only, then the integral over $d\omega$ is very small, too. The integral over solid angle can be approximated numerically as a finite sum over many individually small solid angles, only one of which has the size and is in the direction of the Sun. For solar radiation, then the integral (3.1) reduces to:

$$I = \sum_{i=1}^{N_{\text{eq}}} E_i \cos \varphi_i d\omega_i = E_m \ d\omega_m$$

(3.4)

Consequently, one can dispense with the solid angle integration and just use $I$ to characterize direct solar radiation. However, terrestrial radiation (infra-red) and scattered solar radiation come from many angles and integration over solid angles and using radiance $E$ are required.

Planck’s law of blackbody radiation defines the monochromatic radiance, $E_\lambda$ (or spectral radiance) as the amount of radiant energy flux passing through a given area at a wavelength ($\lambda$) of electromagnetic radiation for a given blackbody temperature ($T$) from a specific direction. The units of $E_\lambda$ are commonly $W/(sr \ m^2 \ nm)$ where the length scale nm has scale similar to the wavelength of the radiation $\lambda$. Planck’s law may be written

$$E_\lambda = \frac{2c_b c^2}{\lambda^5 \left\{ \exp \left( \frac{c_c}{c_b \lambda T} \right) - 1 \right\}}$$

(3.5)

where $T$ is the temperature of the black body emitter, lambda ($\lambda$) is the wavelength of light, $c_h = 6.625 \times 10^{-34}$ Js is the Planck constant, $c = 3 \times 10^8$ m/s is the speed of light, $c_b = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant.
Planck’s law contains two functions of $\Lambda$. As $\Lambda$ becomes smaller, $\Lambda^5$ decreases but the term in the curly brackets becomes larger more rapidly making (3.5) decrease to zero as $\Lambda$ goes to zero. At very short wavelengths: for decreasing wavelength, the variation with wavelength in square brackets (which is decreasing) dominates the minus fifth power out front (which is increasing). In that case the exponential is much larger than one and an approximate form of (3.5) is:

$$E_{\Lambda} \approx 2c_h c^2 \Lambda^{-5} \exp\left(-\frac{c_h c}{c_B \Lambda T}\right)$$

Relation (3.6) approximates Planck’s law well if $c_h c >> c_B \Lambda T$. For very large values of $\Lambda$ both functions of $\Lambda$ in the denominator of (3.6) become large and so $E_{\Lambda}$ again approaches zero. Therefore, (3.5) indicates that the irradiance must reach a maximum at some wavelength. The location of the maximum is a function of temperature. Taking a derivative of (3.6) w.r.t. $\Lambda$ and setting the result to zero obtains a wavelength $\Lambda_m$ where the emission is a maximum for this approximation. The resultant linear relationship between temperature $T$ and wavelength of maximum emission $\Lambda_m$ is known as Wien’s law:

$$T \Lambda_m = 2.898 \times 10^{-2} \ K m$$

Wien’s law shows how the peak emission moves to a longer wavelength for a cooler temperature of the emitter. Figure 3.3 shows this shift.

Planck’s law also shows that the higher the temperature, the closer the argument of the exp function in (3.5) approaches zero, hence the curly brackets in (3.5) approaches zero, and the larger the values of $E_{\Lambda}$ at all wavenumbers. All these features are evident in Figure 3.3.

The Earth and the surface of the Sun have very different temperatures. Though the Sun has far higher radiance from its surface than Earth, the Sun is far away compared to the distances between emitters and absorbers of terrestrial radiation. Since the net gain of radiant solar energy equals the net emission of terrestrial radiation to space (for energy balance) one can apply a multiplier to the solar spectrum relative to the terrestrial spectra at Earth’s orbit which makes the radiance in the visible wavelengths much higher for the solar than the terrestrial radiance. The same multiplier makes the solar radiation much less than the terrestrial for wavelengths near the maximum of terrestrial radiation. Accordingly, the spectra of $E_{A}$ for the Earth and the Sun (at Earth’s orbit) have very little overlap. Hence, it is customary to assign one part of the
electromagnetic spectrum to solar and one part to terrestrial radiation. The solar radiation is frequently referred to as the “shortwave” radiation while the terrestrial radiation is called the “longwave” radiation. These labels will be used interchangeably.

One can formulate Planck’s law using frequency by first noting that \( c = \lambda \nu \), where \( \nu \) is the electromagnetic frequency at that wavelength. Since \( \nu \) decreases when \( \lambda \) increases an increment of radiance in wavelength is minus an increment in frequency, hence:

\[
E_{\lambda}(\Lambda, T) d\Lambda = -E_{\nu_{\lambda}}(\nu_{\lambda}, T) d\nu_{\lambda}
\]  
(3.8)

Rearranging (3.8) and evaluating \(-d\lambda / d\nu_{\lambda} = c / \nu_{\lambda}^2\) obtains the alternate Planck’s law

\[
E_{\nu_{\lambda}} = \frac{2c_h \nu_{\lambda}^3}{c^2 \exp\left(\frac{c_h \nu_{\lambda}}{c_B T}\right) - 1}
\]  
(3.9)

Integrating (3.9) over all frequencies and over all solid angles finds the total irradiant power per unit area, after use of the identity

\[
\int_{\infty}^{x} \frac{x^3}{\exp(x) - 1} dx = \frac{\pi^4}{15}
\]  
(3.10)

obtains the Stefan-Boltzmann law:

\[
I = \sigma T^4
\]  
(3.11)

where \( I \) here is the total blackbody irradiance and \( \sigma = (2\pi^5 c_p^4) / (15c^2 c_h^3) = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \).

‘Blackbody’ is a descriptive term to visualize the appearance of a ‘perfect’ emitter. According to (3.11), a ‘blackbody’ object emits \( I \) amount of radiant energy per unit area when it has temperature \( T \). This ‘object’ can be many different things: atmospheric gas molecules, snow covered ground, ocean surface, etc. in our applications. However, not all objects are ‘perfect’ emitters.

The ability to emit radiation is measured by a parameter called the emissivity, \( \varepsilon_{\lambda} \). Where \( \varepsilon_{\lambda} \) ranges from 0 (unable to emit radiation at wavelength \( \lambda \)) to 1 (maximum emission). Absorptivity \( (a_{\lambda}) \) is similar to emissivity except that it refers to the ability of that object to absorb
radiation at wavelength $\Lambda$. Kirchhoff’s law states that $a_\Lambda = \varepsilon_\Lambda$. Hence, a good emitter is a good absorber at the same wavelength, similarly for poor emitter. $\varepsilon_\Lambda$ can have quite different values for different wavelengths. For example, the absorptivity of clouds in visible light is low (for a thick cloud, $\varepsilon_\Lambda < \sim 0.2$), but for infrared wavelengths it is much larger (for a thick cloud, $\varepsilon_\Lambda > \sim 0.8$).

3.2.2 Observed Radiation

Planck’s law predicts the radiant emission spectrum from a black body to vary smoothly with electromagnetic wavenumber. The emission spectrum from the Sun only approximately varies as predicted by Planck’s law. The solar spectral irradiance $I_\Lambda$, reaching the Earth is shown as the higher continuous curve in Figure 3.3. For reference, * symbols plot emission values for a blackbody with $T=5620$K, a temperature chosen to have a similar wavelength of maximum emission from Wein’s law. The lower curve is the solar radiation reaching the Earth’s surface for clear sky conditions and the sun directly overhead. Part of the region between these two continuous curves is shaded and estimates the amount of absorption of solar radiation by atmospheric gases; wavelengths with larger amounts of absorption are labeled with the primary absorbing gas at those wavelengths. The clear area between the two continuous curves is an estimate of the amount of scattered solar radiation. The scattering is comparatively small for the infrared wavelengths (0.8 – 1000 $\mu$m) but becomes much stronger in the near-ultraviolet ($\sim 0.3$ - 0.4 $\mu$m). The human eye cannot see ultraviolet light; the closest wavelengths it can see are perceived as the color purple ($\sim 0.43$ $\mu$m). While the fractional scattering is greater for purple, the solar radiance is greater for blue ($\sim 0.45$ – 0.5 $\mu$m). Scattering is a type of reflection, where the incoming light is reflected into different directions. For clean air, these reflections are uniformly distributed in different directions. This strong scattering is why the sky is a rather uniform blue at midday. (Many other factors influence the actual sky color such as the presence of pollutants, the longer path through the air in directions close to the horizon, etc.)

The longwave terrestrial radiation varies greatly from place to place due to temperature variation of the primary emitting surface. The primary emitting surface varies with the wavelength of electromagnetic emission. For some wavelengths the atmosphere is nearly transparent and emission is from the Earth’s surface. Temperature varies considerably at the surface and with altitude in the atmosphere. The emission from high mountains and cold polar regions differs from hot subtropical desert surfaces. If thick clouds are present, the infrared emission is primarily from the tops of those clouds. For other wavelengths the cloud-free
atmosphere absorbs strongly and emission is primarily from a level in the atmosphere and temperature varies strongly with atmospheric elevation. The level of primary emission is related to the concentration of the absorber along the path followed by the radiation, as will be shown later in this chapter (when radiative-convective models are described). For the discussion here, it is sufficient to know that wavelengths strongly absorbed by the atmosphere will, from Kirchhoff’s law also be equally strongly emitted by the atmosphere. From Wien’s law the maximum emission by a black body for terrestrial surface and atmospheric temperatures occurs at infrared wavelengths. Hence, the atmospheric gases that absorb radiation strongly in the infrared wavelengths, sometimes called ‘greenhouse gases’, are the most important for terrestrial radiative balance. The primary greenhouse gas is water vapor (H₂O). The next most important greenhouse gas is carbon dioxide (CO₂). There are other gases with minor, but notable absorption, such as ozone (O₃) and methane (CH₄).

Figure 3.3b shows the terrestrial radiance spectrum measured by a polar orbiting satellite looking down as it passed over Guam island at a specific date and time. Because temperature varies over the surface, the spectrum looks different measured from a polar region (generally lower values at most wavelengths) and from a hot desert region (generally higher values at most wavelengths). These differences might be estimated from the smooth lines plotted on the figure show blackbody radiance spectra for different emitter temperatures. Though the actual emission varies from place to place, there are some consistencies that are emphasized here.

Figure 3.3b indicates higher radiance values in the wavelengths from about 8 to 14 \( \mu \text{m} \). This range of wavelengths is sometimes called the primary ‘atmospheric window’. The name makes clear that emission at these wavelengths largely passes through the atmosphere without absorption. Consequently, the radiance reflects surface temperatures (~295K in this figure, a reasonable temperature near the surface at this locale). There is a secondary and less transparent atmospheric window for wavelengths of about 16 to 22 \( \mu \text{m} \). The figure also shows wavelengths for which specific absorbing gases are more prominent, such as ozone from 9.4 to 9.8 \( \mu \text{m} \), and both CO₂ and H₂O near 14-16 \( \mu \text{m} \). Water vapor is the strongest greenhouse gas based on the prevalence of wavelengths absorbed and the amount of drop in radiance. (Water vapor is also a strong absorber of shortwave irradiance, as deduced from figure 3.3a.) Where there is strong absorption, the main emission is effectively from a higher and thus colder atmospheric level. The blackbody emission curves illustrate Wien’s law, with the maximum emission occurring at longer wavelengths for colder temperatures. Also notable in passing is the blackbody emission is less at all wavelengths for a colder temperature.

The units in figure 3.3a differ from 3.3b as does the magnitude. To discuss energy balance, it is useful to deduce the total amount of solar radiant energy reaching the Earth (known
as the ‘solar constant’). The solar constant is both an integral over the electromagnetic wavelengths of the solar irradiance, but also must account for the distance between the Sun and Earth. That Earth-Sun distance defines a small solid angle on a sphere whose radius is the Earth-Sun distance. One can estimate the solar spectral irradiance from the solar blackbody radiance (3.5) times the ratio of the distances (Sun radius over one astronomical unit) squared times pi (to obtain solid angle and thus irradiance). The result for 6000K temperature is plotted in figure 3.3a. One can estimate the solar constant ($I_{\text{Sol}}$, solar irradiance at the Earth’s orbit) from the Stefan-Boltzmann law (3.11) times that same ratio squared times pi. Estimates of the solar constant vary between 1356 and 1370 W/m$^2$ (see Trenberth et al., 2009, for a review). The estimate for 6000K is $I_{\text{Sol}} = 1595$ W/m$^2$ which is too high, hence an effective blackbody temperature of the Sun is lower (~5772K) for $I_{\text{Sol}} = 1365$ W/m$^2$ used here.

The atmospheric absorption of the electromagnetic radiation has extremely important implications for the planet. Both the absorption and the scattering of the solar radiation at very short wavelengths ($\leq 0.4 \mu$m) greatly favors many complex life forms on the planet. The high energy of ultraviolet light is known to damage many plants and may adversely affect the health of animals. (Life is relevant to our discussion since life is necessary to maintain the atmosphere’s present chemical composition. Plants significantly alter the surface budgets of momentum, energy, and moisture, too.) Absorption of longwave radiation also greatly favors the development of life on Earth by elevating the temperature of the Earth and its atmosphere. Without the longwave atmospheric absorption, the Earth’s surface would be much cooler. Instead, the average temperature of the Earth’s surface is kept high enough to permit liquid water to occur. (This longwave absorption is sometimes erroneously referred to as the “greenhouse effect”; in actuality a greenhouse builds elevated temperatures by trapping air in a confined space.)

Solar spectral irradiance reaching the Earth is reflected, transmitted, and absorbed by the atmosphere and reflected and absorbed by the surface, in complex ways. Similarly, the longwave emission from the Earth’s surface and atmosphere (including clouds) is also complex. A starting point for discussion the atmospheric radiation is to consider energy balance between solar energy gain and infrared loss to space.

The global radiative energy balance is diagrammed schematically in Figure 3.4. Budgets for the earth’s surface, the atmosphere, and outer space are given. Of course, the total amount absorbed or emitted must balance, otherwise the earth or atmosphere would experience a net heating up or cooling down. As anticipated above, Figure 3.4 partitions the energy into shortwave and longwave components. The presentation includes both actual irradiances as well as their percentage of the total shortwave energy encountering the top of the atmosphere. That
total amount reaching the TOA varies with latitude, season, and time of day. However, a global balance is shown first. In this global balance the amount of radiation intercepted by the Earth equals its cross sectional area. The Earth rotates on its axis spreading the interception over the entire Earth (at the equinoxes). The surface area of the Earth is four times the cross sectional area, hence the global incoming solar irradiance is \( I_S = \frac{I_{sol}}{4} \sim 341 \text{ W/m}^2 \).

Figure 3.4 summarizes the global energy balances for the Earth system, where separate balances are shown for the Earth and space, for the whole atmosphere, and for the Earth’s surface. The figure separates the partitioning of solar energy input on the left from the terrestrial energy elements on the right. Various authors have estimated these elements and a summary can be found in the primary reference used for this figure (Trenberth et al., 2009). The numbers are challenging to obtain with precision and some numbers are estimated; it is customary to adjust the numbers so that there is a balance for each part of the Earth system. Trenberth et al. (2009) chose not to make a full adjustment and leave a net absorption of 0.9 Wm\(^{-2}\) energy gain (from Hansen et al., 2005) by the Earth. Hence, the primary purpose of this diagram resides not in the precise numbers but to illuminate the relative proportionalities of the elements shown. To facilitate illustrating the proportions, both the unitary value and the percentage of the solar input (I\(_S\)) are indicated.

The shortwave radiation has these global properties in Figure 3.4: about 30% of I\(_S\) is reflected back to space; this reflection is labeled the albedo. Much of the reflection is from clouds (18%) while about 5% is the scattering (primarily of blue and purple wavelengths of sunlight). Only about half of I\(_S\) reaches the surface and a small fraction, about 12-13% of that is reflected back to space. While snow covered surfaces and deserts have high albedo, most of the Earth is covered by dark oceans and forests having small albedo. The remaining fraction of sunlight, less than a quarter of I\(_S\), is absorbed by the atmosphere and its clouds.

The Earth’s surface absorbed 47% of solar radiant energy but also a larger amount (energy equivalent to 98% of I\(_S\)) emitted downward by the atmosphere. Earth’s surface loses energy by three means: infrared emission, evaporating water (surface latent heat fluxes, LHF), and surface sensible heat fluxes (SHF). The atmosphere also gained energy, from absorbing sunlight, from absorbing longwave emission by the Earth’s surface, plus the LHF and SHF.

Examining the heat fluxes finds the latent heat fluxes to be 4-5 times as large as the sensible heat fluxes. The atmosphere gains energy from the sensible heating at its lowest levels, by conduction and small scale turbulent mixing leading to convective thermals or mechanical mixing into higher altitudes. The energy from the latent heat flux is mixed vertically in a similar manner however that energy is not released into heating the air until the moisture condenses. The condensation can occur a great distance from the location of the latent heat surface flux.
Condensation of the moisture input by those latent heat fluxes also plays a major role in the general circulation; for example, it is the major driving mechanism of the Hadley circulation (Chapter 10).

The emission from the tops of clouds (equivalent to 9% of IS) has a significant role in the diabatic energy conversions (Chapter 7) and the diabatic forcing of the zonal mean meridional circulations (Chapter 10).

Excluding the solar radiation reflected and scattered by the atmosphere as not available to heat the Earth system, the atmosphere is more transparent to shortwave radiation (54/77*100=70% passes through) than to longwave radiation (40/396=10% passes through). Simple models (next) show that the atmosphere being more transparent to shortwave radiation than to longwave radiation elevates the temperatures of the Earth’s surface above expectations based only on absorbed solar energy, a property sometimes called the ‘greenhouse effect’. The effect is evident in Figure 3.4 as the large downward and upward longwave emissions between atmosphere and surface.

A simple analogy using a ping pong ball to represent a unit of radiant energy may be useful in visualizing the effect. The solar radiation absorbed by the earth is represented by the impact of a ping pong ball as it is thrown toward the floor, one ball each minute. The floor represents the earth, so that the upward rebound of the ball represents the net infrared emission from the earth. The atmosphere might be thought of as a table. When tossed downward, the ball bounces off the floor, then off the underside of the table, and again off the floor before it is caught, that last representing exit to space. In this trivial model, the radiant energy received by the “Earth” is twice the incoming value because the ball must bounce off the floor twice before it is caught.

A simple quantitative model is the ‘glass slab’ model, versions of which can be found in various sources. ‘Glass’ in this context highly simplifies the atmosphere as a substance which is more transparent in visible than in infrared wavelengths and since it is a solid, so no effects from an ‘atmospheric’ circulation are included. The albedo (A) from reflection and scattering off the atmosphere and off the surface is already removed to specify the shortwave radiation input (IN). Similarly, the longwave emissivity of the surface is assumed to be 1, i.e. the surface is a ‘blackbody’. Figure 3.5 describes the model components, where subscripts designate: ‘A’ for atmosphere, ‘G’ for surface, ‘S’ for shortwave, and ‘L’ for longwave. The longwave emission from the atmospheric ‘glass slab’ is in all directions upward and all directions downward while the corresponding emission from the surface is only all directions upward. IS is as before. Irradiance from the surface and from the atmosphere are uniquely determined from radiative balance of the surface and atmosphere once the four absorptivities are specified. From those
irradiances, the Stefan-Boltzmann law, with appropriate emissivity, obtains surface and atmospheric temperatures.

Radiative balance for the atmosphere is

$$a_{SA} I_{IN} + a_{LA} I_{G} = 2a_{LA} I_{A}$$  \hspace{1cm} (3.12)

Radiative balance for the surface is

$$a_{SG} I_{IN} + a_{LA} I_{A} = I_{G}$$  \hspace{1cm} (3.13)

The $I_A$ as used in (3.12) and (3.13) is the black body irradiance because the grey body emissivity of the atmosphere ($a_{LA}$) is included explicitly in these equations and in the corresponding figure. This pair of equations can be solved to obtain

$$a_{LA} I_{A} = I_{IN} \left( \frac{a_{SA} + a_{LA} - a_{SA} a_{LA}}{2 - a_{LA}} \right)$$  \hspace{1cm} (3.14)

And

$$I_{G} = I_{IN} \left( \frac{2 - a_{SA}}{2 - a_{LA}} \right)$$  \hspace{1cm} (3.15)

To obtain the temperatures of atmosphere and surface, solve for $T_A$ and $T_G$ from

$$I_{G} = \sigma T_G^4$$  \hspace{1cm} (3.16)

And

$$I_{A} = \sigma T_A^4$$  \hspace{1cm} (3.17)

The ‘greenhouse’ effect is clear from (3.15) that if $a_{LA} > a_{SA}$ then $I_{G} > I_{IN}$ and the irradiance, and therefore the temperature, is greater than if it were based solely on the solar irradiance absorbed. The extremes of (3.15) are instructive. If the atmosphere were completely transparent to shortwave and completely opaque to longwave radiation, then the surface irradiance would be twice the solar input. For values in Figure 3.4, since $a_{SA} = 0.33$ and $a_{LA} = 0.9$ the factor multiplying $I_{IN}$ in (3.15) is $\sim 1.52$. Similarly, Figure 3.4 implies that $I_{IN} = 239 \text{ W/m}^2$ making $I_{G} = 363 \text{ W/m}^2$ and $T_G = 283 \text{ K}$. Solving for the atmosphere, $I_{A} = 0.9424 \ I_{IN} = 225.24 \text{ W/m}^2$ and $T_A = 251 \text{ K}$. For comparison, if there was no atmosphere, with the same albedo $A=0.3$, then $I_{G} = 239 \text{ W/m}^2$ and $T_G = 255 \text{ K}$.

A third simple model can incorporate LHF and SHF and obtain atmospheric and surface irradiances and temperatures via an infinite series.

REVISIONS ONLY TO HERE
Figure 3.2
Figure 3.3 (a) Solar spectral irradiance (shortwave radiation) Top of atmosphere (TOA) and surface spectral values based on data provided by R. Rohde (2007; http://www.globalwarmingart.com/wiki/File:Solar_Spectrum.png). Shaded area estimates the terrestrial absorption based on Thakaekara (1976). Asterisks are point values for spectral irradiance at the Earth’s orbit if the sun were a 6000K black body. (b) Terrestrial radiation for a region of the subtropical Pacific measured by a polar orbiting satellite as it passed over Guam. This panel redrawn from data in Liou (1980). The abscissa is equally-spaced in wavenumber, but stretched in wavelength.
Figure 3.4 Estimates of major components of the global energy balance based upon data in Trenberth et al. (2009) with adjustments in Grotjahn (1993). Two numbers are presented, with the top number in units of W/m² while the number in parentheses is the corresponding percentage of the incoming radiation. Abbreviations: LHF is latent heat flux from the Earth’s surface; SHF is sensible heat flux from the Earth’s surface; NA is net absorption by the Earth system (ocean, land, atmosphere) as estimated by Hansen et al. (2005).
Figure 3.5 Parameters for the ‘glass slab’ model of radiative balance for the global atmosphere.